

## Excel Solutions to the Chemical Engineering Problem Set

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### **Introduction**

These solutions are to the problems given in Reference (1) which were presented at the ASEE Chemical Engineering Summer School held in Snowbird, Utah on August 13, 1997.

Excel version 7.0 (part of Microsoft's Office 95) was used to solve all the problems. Considerable use was made of Visual Basic for Applications which is part of Excel. User defined function were written (in VBA) to carry a number of functions including the numerical integration of differential equations (4th order Runge-Kutta method). Use was also made of the Solver and Goal Seek programs routines that are supplied as add-ins with Excel 7.0.

**Excel Problem 1 Solution****(a)**

Use was made of Goal Seek to find the value of V to force the difference between the right and left sides of Ex\_Eq (1-1) to zero.

$$(P + a/V^2)(V-b) - RT = 0 \quad \text{Ex\_Eq (1-1)}$$

where

$$a = (27/64) (R^2 T_c^2 / P_c) \quad \text{Ex\_Eq (1-2)}$$

$$b = (RT_c) / (8P_c) \quad \text{Ex\_Eq (1-3)}$$

**(b) and (c)**

Changing the value of Pr changes the value of  $P = Pr \cdot P_c$ . The values of Z and V determined at the solution were recorded on EX Spreadsheet (1-1).

**EX Spreadsheet (1-1)**

Problem 1

Molar Volar and Compressibility Factor from Van der Waals Equation

	Value	P	Preduced	V	Z
P = Pressure in atm=Pr*Pc	2226				
V = molar volume in liters/g-mol	0.046175	55.9839	0.503	0.575084	0.871819
T= temperature in K	450	111.3	1	0.233516	0.703829
R= gas constant atm-liter/g-molK	0.08206	222.6	2	0.077266	0.465765
Tc= citical temp for ammonia	405.5	445.2	4	0.060654	0.731259
Pc=critical pressure for ammonia	111.3	1113	10	0.050876	1.533418
		2226	20	0.046175	2.783489
Preduced	20				
Z=compressilby factor=Pv/RT	2.783489				
a	4.196946				
b	0.037371				
Value of RT	36.927				
Value of P+a/V^2	4194.415				
Value of V-b	0.008804				
LHS	36.92738				
f(V)=0	0.00038				

**Excel Problem 2 Solution****(a)**

Direct use was made of function MINVERSE to find the inverse of the matrix of linear equations. Then function MMULT was used to multiply the inverse by the right side of the equations yielding the flows of streams D1, B1, D2 and B2.

**(b)**

After calculation of the total flows of streams B and D their compositions were calculated from the material balance on each component. See EX Spreadsheet (2-1).

To use both MINVERSE and MMULT the output area had to be selected first and **Ctrl+Shift+Enter** used after specifying the name of the function and the input argument ranges.

**EX Spreadsheet (2-1)**

Problem 2

Steady State Material Balances on a Separation Train

Material balance Equation matrix

0.07	0.18	0.15	0.24
0.04	0.24	0.1	0.65
0.54	0.42	0.54	0.1
0.35	0.16	0.21	0.01

Inverse from Use of MINVERSE

RHS

Soln from Use of  
MMULT  
Molar Flows

-2.81048387	1.221774	-1.8024194	6.060484	10.5	D1	26.25
94.3629032	-29.8306	-36.685484	41.1371	17.5	B1	17.5
-66.0443548	20.24597	30.528226	-36.2056	28	D2	8.75
-24.5080645	9.362903	8.9596774	-9.99194	14	B2	17.5

Calculation from Results (Part b)

Molar Flows	Component	Composition B	Composition D
B=D2+B2	Xylene	0.2100	0.1140
D=D1+B1	Styrene	0.4667	0.1200
	Toluene	0.2467	0.4920
	Benzene	0.0767	0.2740

### Excel Problem 3 Solution

(a), (b) and (c)

The spreadsheet was set up with the data for temperature and pressure entered in the indicated columns. Columns for Temp in Deg K,  $1/T_k \times 1000$  and Log P were developed

Columns were then added for each of the curve fits that were to be carried out along with the parameters that would be manipulated to generate the calculated curves. These are shown above the calculated columns as parameters.

Finally a row of sum of squares cells were added along the bottom. The contents were set up as array formulas and entered with Crtl+Shift+Enter. For example for the Calculated Clausius-Clapeyron column: in the sum of squares cell the following was entered:

$$\text{SUM}((F16:F25-E16:E25)^2)$$

followed by **Crtl+Shift+Enter**.

The Solver program was then utilized for each of the curve fits. For example for the Clausius Clapeyron fit F5 and F6 (A and B) were manipulated until F28 (the sum of squares) was minimized. Forward differences were used to obtain the derivatives.

For curve fitting the polynomials, the starting values were taken as the lower polynomials ending values. To gain greater accuracy, derivatives were taken as central differences.

### **EX Spreadsheet (3-1)**

Problem 3  
Vapor Pressure Data Representation By Polynomials and Equations

Parameters

A and a0	8.75200927	5.76469991	64.4059872	-0.5819143	24.459324	24.6788	24.754264
B and a1	2035.33113	676.233806	5.89071425	2.06714948	1.1981007	1.6062	1.6090165
C and a2		153.786135		0.08615248	0.0394481	0.0360443	0.0356053
D and a3					0.0007449	0.00041312	0.000413
E and a4						3.96E-06	4.23E-06
F and a5							-2.51E-09

Temp. Deg C	Temp. Deg K	$1/T_k \times 1000$	Press, P mm Hg	Log P	Calculated Clausius-Clap Log P	Calculated Antoine Log P	Calculated Polynomial Power 1 Pressure	Calculated Polynomial Power 2 Pressure	Calculated Polynomial Power 3 Pressure	Calculated Polynomial Power 4 Pressure	Calculated Polynomial Power 5 Pressure	
-36.7	236.45	4.229223937		1	0	0.14414	-0.01082	-151.78323	39.592	-3.200	1.048	1.079
-19.6	253.55	3.943995267		5	0.699	0.72467	0.72518	-51.05201	-8.002	10.522	4.518	4.417
-11.5	261.65	3.821899484		10	1	0.97318	1.01207	-3.33723	-12.960	14.765	10.415	10.406
-2.6	270.55	3.696174459		20	1.301	1.22907	1.29184	49.09013	-5.374	21.598	20.739	20.804
7.6	280.75	3.561887801		40	1.6021	1.50239	1.57454	109.17542	20.105	36.170	39.162	39.235
15.4	288.55	3.465603881		60	1.7782	1.69836	1.76772	155.12299	51.684	54.986	59.694	59.721
26.1	299.25	3.341687552		100	2	1.95057	2.00547	218.15363	112.059	95.846	100.339	100.278

42.2	315.35	3.171079753	200	2.301	2.29781	2.31428	312.99413	240.076	201.251	200.265	200.165
60.6	333.75	2.996254682	400	2.6021	2.65364	2.61042	421.38327	441.070	407.708	399.768	399.869
80.1	353.25	2.830856334	760	2.8808	2.99028	2.87340	536.25220	717.752	756.353	760.052	760.026
Sum of Squares					0.060732441	0.00223062	118590.556	8517.49512	204.73337	1.98960209	1.9436128
Variance (sum of Squares/degrees of freedom)					0.007591555	0.00031866	14823.8195	1216.78502	34.122228	0.39792042	0.4859032

**Excel Problem 4 Solution**

EX Spreadsheet (4-1) was set up so that the problem could be solved using Solver. The objective function was taken (arbitrarily) as the sum of squares of the three equilibrium equations:

$$K_{C1} * C_A * C_B - C_C * C_D = 0 \quad \text{EX Eq (4-1)}$$

$$K_{C2} * C_B * C_C - C_X * C_Y = 0 \quad \text{EX Eq (4-2)}$$

$$K_{C3} * C_A * C_X - C_Z = 0 \quad \text{EX Eq (4-3)}$$

Values of  $C_A$ ,  $C_B$ ,  $C_C$  and  $C_Y$  were computed from the three unknowns  $C_D$ ,  $C_X$  and  $C_Z$  as given in Reference (1) - Eq (12).

**(a) and (b)**

Solver found a feasible solution in both cases i.e. starting the unknowns equal to zeros and one.

The solutions were copied (using paste special) into the solution matrix. Solution (b) however having a negative composition is not physically real.

**(c)**

Solver could not find a feasible solution starting with unknowns = 10.

**EX Spreadsheet (4-1)**

Problem 4  
Reaction Equilibrium for Multiple Gas Phase Reactions

Parameters and Unknowns		Obj	1.15E-12		
	Value				
CA	0.420688	Eq EX 4-1	-8.37E-07		
CB	0.242896	Eq EX 4-2	-5.4E-07		
CC	0.153566	Eq EX 4-3	3.91E-07		
CD	0.705335 Unk				
CX	0.177793 Unk				
CY	0.551769				
CZ	0.373977 Unk				
Solutions to the Chemical Equilibrium Problem					
CAO	1.5	Part (a)	Part (b)	Part (c)	
CBO	1.5	CD	0.705335	0.055556	
KC1	1.06	CX	0.177793	0.597219	Feasible
KC2	2.63	CZ	0.373977	1.082074	Solution
KC3	5	CA	0.420688	0.362371	Could
		CB	0.242896	-0.234849	Not
		CC	0.153566	-1.623738	Be
		CY	0.551769	1.679293	Found

**Excel Problem 5 Solution**

The spreadsheet was set up to use Goal Seek to find the value of  $v_t$  that satisfies the following equation:

$$v_t^2 * 3 * CD * \rho - 4 * g * (\rho_p - \rho) * D_p = 0 \quad \text{EX Eq (5-1)}$$

A VBA user defined function CD value (EX Listing (5-1)) was written to evaluate CD as a function of Re.

**(a)**

Goal Seek was used to find  $v_t$  when  $g = 9.80665 \text{ m/s}^2$  (Factor = 1) in EX Eq (5-1).

**(b)**

Goal Seek was used to find  $v_t$  when the acceleration is 30g (Factor = 30).

**EX Spreadsheet (5-1)**

Problem 5

Terminal Velocity of Falling Particles

Parameters/Variables

	(b)	(a)
Factor	30	1
$v_t$ -m/s	0.20602151	0.0157816
$\rho$ kg/m <sup>3</sup>	994.6	994.6
$g$ - m/s <sup>2</sup>	294.1995	9.80665
$\rho_{hop}$ - kg/m <sup>3</sup>	1800	1800
$D_p$ - m	0.00020800	0.00020800
$v_{ls}$ -kg/m-s	0.00089310	0.00089310
Re	47.7226196	3.6556427
CD	1.55661834	8.8426498
EX Equation (5-1)	5.4431E-05	9.071E-06

### Excel Problem 6 Solution

User defined functions rk4a and fff1..fff5 were written to carry out the integration of the three ordinary differential equations as given in Reference (1) Eq 20-22. The user defined functions are given in EX Listing (6-1) and EX Listing (6-2) and the way the user defined array functions are called (entered after the output cells were selected) is given in EX Listing (6-3).

EX Spreadsheet (6-1) gives the steady state values of T1, T2 and T3 as approximately 30.95 C, 41.37 C and 51.27 C. A chart of the values is given in EX Figure (6-1). From the spreadsheet it takes about 62 min to achieve 99% of the steady state value of T3 (the slowest changing temperature).

#### EX Spreadsheet (6-1)

Problem 6					24	12	27.84569	31.58976	32.91872
Heat Exchange in a Series of Tanks					25	12.5	28.00459	31.94326	33.39142
Parameters					26	13	28.15535	32.28622	33.8569
W kg/min	100				27	13.5	28.29841	32.6188	34.31502
Cp KJ/kg	2				28	14	28.43415	32.94115	34.76566
TO C	20				29	14.5	28.56295	33.25345	35.20871
UA	10				30	15	28.68516	33.55589	35.64406
KJ/min C					31	15.5	28.80111	33.84866	36.07165
Tsteam	250				32	16	28.91114	34.13196	36.49139
C					33	16.5	29.01554	34.40599	36.90323
M kg	1000				34	17	29.1146	34.67096	37.30713
h min	0.5				35	17.5	29.2086	34.92707	37.70307
					36	18	29.29778	35.17455	38.09101
					37	18.5	29.38241	35.41359	38.47097
					38	19	29.46271	35.64443	38.84293
					39	19.5	29.5389	35.86727	39.20691
					40	20	29.61119	36.08233	39.56294
					41	20.5	29.67979	36.28982	39.91105
					42	21	29.74488	36.48995	40.25127
					43	21.5	29.80663	36.68293	40.58367
					44	22	29.86523	36.86897	40.9083
					45	22.5	29.92084	37.04828	41.22522
					46	23	29.9736	37.22105	41.53451
					47	23.5	30.02366	37.38749	41.83623
					48	24	30.07116	37.5478	42.13048
					49	24.5	30.11623	37.70215	42.41734
					50	25	30.15899	37.85076	42.69691
					51	25.5	30.19957	37.99379	42.96927
					52	26	30.23807	38.13142	43.23455
					53	26.5	30.27461	38.26385	43.49282
					54	27	30.30927	38.39124	43.74422
					55	27.5	30.34217	38.51375	43.98884
					56	28	30.37338	38.63156	44.22681
					57	28.5	30.40299	38.74483	44.45822
					58	29	30.43109	38.85371	44.68321
					59	29.5	30.45775	38.95835	44.9019
					60	30	30.48305	39.05891	45.11439
					61	30.5	30.50705	39.15552	45.32082
					62	31	30.52983	39.24833	45.5213
					63	31.5	30.55144	39.33747	45.71595
					64	32	30.57195	39.42308	45.90491
					65	32.5	30.5914	39.50528	46.08829
					66	33	30.60987	39.58421	46.26621
Index	Time (min)	T1 (C)	T2 (C)	T3 (C)					
0	0	20	20	20					
1	0.5	20.56017	20.57405	20.57428					
2	1	21.09168	21.14531	21.14708					
3	1.5	21.59602	21.71258	21.71833					
4	2	22.07455	22.27475	22.28786					
5	2.5	22.52862	22.83088	22.85551					
6	3	22.95946	23.38011	23.42104					
7	3.5	23.36826	23.92168	23.98422					
8	4	23.75615	24.45495	24.54478					
9	4.5	24.12421	24.97935	25.10242					
10	5	24.47344	25.49439	25.65686					
11	5.5	24.80481	25.99966	26.20779					
12	6	25.11923	26.4948	26.7549					
13	6.5	25.41757	26.97954	27.2979					
14	7	25.70065	27.45364	27.83648					
15	7.5	25.96926	27.91692	28.37034					
16	8	26.22412	28.36925	28.89919					
17	8.5	26.46595	28.81054	29.42276					
18	9	26.69541	29.24073	29.94077					
19	9.5	26.91314	29.6598	30.45297					
20	10	27.11973	30.06777	30.95911					
21	10.5	27.31575	30.46468	31.45896					
22	11	27.50175	30.85058	31.95231					
23	11.5	27.67824	31.22557	32.43896					

67	33.5	30.62739	39.65997	46.4388	126	63	30.9377	41.2621	50.80358
68	34	30.64401	39.73269	46.60617	127	63.5	30.93845	41.26759	50.82409
69	34.5	30.65978	39.80247	46.76846	128	64	30.93917	41.27285	50.84382
70	35	30.67474	39.86944	46.92577	129	64.5	30.93984	41.27787	50.86278
71	35.5	30.68894	39.93369	47.07824	130	65	30.94048	41.28266	50.88102
72	36	30.70242	39.99533	47.22597	131	65.5	30.94109	41.28724	50.89855
73	36.5	30.7152	40.05446	47.36908	132	66	30.94167	41.29161	50.9154
74	37	30.72733	40.11116	47.5077	133	66.5	30.94222	41.29579	50.9316
75	37.5	30.73884	40.16555	47.64193	134	67	30.94274	41.29978	50.94717
76	38	30.74976	40.2177	47.77189	135	67.5	30.94323	41.30359	50.96214
77	38.5	30.76013	40.26769	47.89769	136	68	30.9437	41.30723	50.97651
78	39	30.76996	40.31563	48.01944	137	68.5	30.94414	41.3107	50.99033
79	39.5	30.77929	40.36158	48.13724	138	69	30.94456	41.31402	51.00361
80	40	30.78814	40.40562	48.25122	139	69.5	30.94496	41.31719	51.01636
81	40.5	30.79654	40.44783	48.36146	140	70	30.94534	41.32021	51.02861
82	41	30.80451	40.48827	48.46808	141	70.5	30.9457	41.3231	51.04038
83	41.5	30.81208	40.52703	48.57117	142	71	30.94604	41.32586	51.05169
84	42	30.81925	40.56416	48.67084	143	71.5	30.94637	41.32849	51.06255
85	42.5	30.82606	40.59974	48.76718	144	72	30.94668	41.33101	51.07298
86	43	30.83252	40.63382	48.86029	145	72.5	30.94697	41.33341	51.08299
87	43.5	30.83865	40.66646	48.95027	146	73	30.94724	41.3357	51.09261
88	44	30.84447	40.69772	49.03719	147	73.5	30.94751	41.33788	51.10184
89	44.5	30.84999	40.72766	49.12116	148	74	30.94776	41.33997	51.11071
90	45	30.85523	40.75634	49.20227	149	74.5	30.94799	41.34196	51.11922
91	45.5	30.86019	40.78379	49.28059	150	75	30.94822	41.34387	51.12739
92	46	30.86491	40.81007	49.35621	151	75.5	30.94843	41.34568	51.13524
93	46.5	30.86938	40.83524	49.42922	152	76	30.94863	41.34741	51.14276
94	47	30.87363	40.85933	49.49969	153	76.5	30.94882	41.34907	51.14999
95	47.5	30.87766	40.88239	49.56771	154	77	30.94901	41.35065	51.15693
96	48	30.88148	40.90446	49.63335	155	77.5	30.94918	41.35215	51.16358
97	48.5	30.8851	40.92558	49.69668	156	78	30.94934	41.35359	51.16997
98	49	30.88855	40.94579	49.75778	157	78.5	30.9495	41.35496	51.1761
99	49.5	30.89181	40.96514	49.81672	158	79	30.94965	41.35627	51.18198
100	50	30.89491	40.98365	49.87356	159	79.5	30.94979	41.35752	51.18762
101	50.5	30.89785	41.00136	49.92838	160	80	30.94992	41.35871	51.19303
102	51	30.90064	41.0183	49.98124	161	80.5	30.95004	41.35985	51.19822
103	51.5	30.90328	41.03451	50.0322	162	81	30.95016	41.36093	51.20321
104	52	30.90579	41.05002	50.08133	163	81.5	30.95028	41.36197	51.20798
105	52.5	30.90818	41.06485	50.12868	164	82	30.95038	41.36295	51.21257
106	53	30.91044	41.07903	50.17432	165	82.5	30.95049	41.36389	51.21696
107	53.5	30.91258	41.0926	50.2183	166	83	30.95058	41.36479	51.22118
108	54	30.91462	41.10558	50.26068	167	83.5	30.95068	41.36565	51.22522
109	54.5	30.91655	41.11799	50.3015	168	84	30.95076	41.36647	51.2291
110	55	30.91838	41.12985	50.34083	169	84.5	30.95085	41.36725	51.23281
111	55.5	30.92012	41.1412	50.37872	170	85	30.95092	41.36799	51.23638
112	56	30.92177	41.15204	50.4152	171	85.5	30.951	41.3687	51.2398
113	56.5	30.92334	41.16242	50.45034	172	86	30.95107	41.36938	51.24307
114	57	30.92482	41.17233	50.48418	173	86.5	30.95114	41.37002	51.24621
115	57.5	30.92623	41.18181	50.51675	174	87	30.9512	41.37064	51.24922
116	58	30.92757	41.19087	50.54811	175	87.5	30.95126	41.37123	51.25211
117	58.5	30.92884	41.19953	50.5783	176	88	30.95132	41.37179	51.25488
118	59	30.93004	41.20781	50.60736	177	88.5	30.95137	41.37232	51.25753
119	59.5	30.93118	41.21572	50.63532	178	89	30.95142	41.37283	51.26007
120	60	30.93227	41.22328	50.66223	179	89.5	30.95147	41.37332	51.26251
121	60.5	30.9333	41.23051	50.68813	180	90	30.95152	41.37378	51.26484
122	61	30.93427	41.23741	50.71304	181	90.5	30.95156	41.37422	51.26708
123	61.5	30.9352	41.24401	50.73701	182	91	30.95161	41.37464	51.26922
124	62	30.93608	41.25032	50.76007	183	91.5	30.95164	41.37505	51.27127
125	62.5	30.93691	41.25634	50.78225	184	92	30.95168	41.37543	51.27324

**EX Listing (6-1) RHS's for rk4a**

'These functions must be modified for actual problem usage

'Written by EMRosen 6/19/97

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'Departments of Chemical Engineering

'prm Array

'prm(1) = W

'prm(2) = Cp

'prm(3) = TO

'prm(4) = UA

'prm(5) = Tsteam

'prm(6) = M

'prm(7) = h

Public Function fff1(x, y1, y2, y3, y4, y5, prm)

fff1 = (prm(1) \* prm(2) \* (prm(3) - y1) + prm(4) \* (prm(5) - y1)) / (prm(6) \* prm(2))

End Function

Public Function fff2(x, y1, y2, y3, y4, y5, prm)

fff2 = (prm(1) \* prm(2) \* (y1 - y2) + prm(4) \* (prm(5) - y2)) / (prm(6) \* prm(2))

End Function

Public Function fff3(x, y1, y2, y3, y4, y5, prm)

fff3 = (prm(1) \* prm(2) \* (y2 - y3) + prm(4) \* (prm(5) - y3)) / (prm(6) \* prm(2))

End Function

Public Function fff4(x, y1, y2, y3, y4, y5, prm)

fff4 = 0

End Function

Public Function fff5(x, y1, y2, y3, y4, y5, prm)

fff5 = 0

End Function

**EX Listing (6-2) rk4a Function**

Public Function rk4a(h, x, y1, y2, y3, y4, y5, prm)

'Written by EMRosen 8/31/97

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'h = step size

'x = independent variable

'y1, y2, y3, y4, y5 = dependent variables

'prm a parameter vector of unspecified length

'kij : i is the k value, j is the equation number or dependent variable

'To implement: must preselect row cells and enter Shift+Ctrl+Enter  
'Passes Entire Array to Spreadsheet

Dim DDD(1 To 5)

k11 = fff1(x, y1, y2, y3, y4, y5, prm)

k12 = fff2(x, y1, y2, y3, y4, y5, prm)

k13 = fff3(x, y1, y2, y3, y4, y5, prm)

k14 = fff4(x, y1, y2, y3, y4, y5, prm)

k15 = fff5(x, y1, y2, y3, y4, y5, prm)

k21 = fff1(x + 0.5 \* h, y1 + 0.5 \* h \* k11, y2 + 0.5 \* h \* k12, y3 + 0.5 \* h \* k13, y4 + 0.5 \* h \* k14, y5 + 0.5 \* h \* k15, prm)

k22 = fff2(x + 0.5 \* h, y1 + 0.5 \* h \* k11, y2 + 0.5 \* h \* k12, y3 + 0.5 \* h \* k13, y4 + 0.5 \* h \* k14, y5 + 0.5 \* h \* k15, prm)

k23 = fff3(x + 0.5 \* h, y1 + 0.5 \* h \* k11, y2 + 0.5 \* h \* k12, y3 + 0.5 \* h \* k13, y4 + 0.5 \* h \* k14, y5 + 0.5 \* h \* k15, prm)

k24 = fff4(x + 0.5 \* h, y1 + 0.5 \* h \* k11, y2 + 0.5 \* h \* k12, y3 + 0.5 \* h \* k13, y4 + 0.5 \* h \* k14, y5 + 0.5 \* h \* k15, prm)

k25 = fff5(x + 0.5 \* h, y1 + 0.5 \* h \* k11, y2 + 0.5 \* h \* k12, y3 + 0.5 \* h \* k13, y4 + 0.5 \* h \* k14, y5 + 0.5 \* h \* k15, prm)

k31 = fff1(x + 0.5 \* h, y1 + 0.5 \* h \* k21, y2 + 0.5 \* h \* k22, y3 + 0.5 \* h \* k23, y4 + 0.5 \* h \* k24, y5 + 0.5 \* h \* k25, prm)

k32 = fff2(x + 0.5 \* h, y1 + 0.5 \* h \* k21, y2 + 0.5 \* h \* k22, y3 + 0.5 \* h \* k23, y4 + 0.5 \* h \* k24, y5 + 0.5 \* h \* k25, prm)

k33 = fff3(x + 0.5 \* h, y1 + 0.5 \* h \* k21, y2 + 0.5 \* h \* k22, y3 + 0.5 \* h \* k23, y4 + 0.5 \* h \* k24, y5 + 0.5 \* h \* k25, prm)

k34 = fff4(x + 0.5 \* h, y1 + 0.5 \* h \* k21, y2 + 0.5 \* h \* k22, y3 + 0.5 \* h \* k23, y4 + 0.5 \* h \* k24, y5 + 0.5 \* h \* k25, prm)

k35 = fff5(x + 0.5 \* h, y1 + 0.5 \* h \* k21, y2 + 0.5 \* h \* k22, y3 + 0.5 \* h \* k23, y4 + 0.5 \* h \* k24, y5 + 0.5 \* h \* k25, prm)

k41 = fff1(x + h, y1 + h \* k31, y2 + h \* k32, y3 + h \* k33, y4 + h \* k34, y5 + h \* k35, prm)

k42 = fff2(x + h, y1 + h \* k31, y2 + h \* k32, y3 + h \* k33, y4 + h \* k34, y5 + h \* k35, prm)

k43 = fff3(x + h, y1 + h \* k31, y2 + h \* k32, y3 + h \* k33, y4 + h \* k34, y5 + h \* k35, prm)

k44 = fff4(x + h, y1 + h \* k31, y2 + h \* k32, y3 + h \* k33, y4 + h \* k34, y5 + h \* k35, prm)

k45 = fff5(x + h, y1 + h \* k31, y2 + h \* k32, y3 + h \* k33, y4 + h \* k34, y5 + h \* k35, prm)

'Set up Results Array

DDD(1) = y1 + (h / 6) \* (k11 + 2 \* k21 + 2 \* k31 + k41)

DDD(2) = y2 + (h / 6) \* (k12 + 2 \* k22 + 2 \* k32 + k42)

DDD(3) = y3 + (h / 6) \* (k13 + 2 \* k23 + 2 \* k33 + k43)

DDD(4) = y4 + (h / 6) \* (k14 + 2 \* k24 + 2 \* k34 + k44)

DDD(5) = y5 + (h / 6) \* (k15 + 2 \* k25 + 2 \* k35 + k45)

rk4a = DDD

End Function

**EX Listing (6-3)**

Cell: A17  
Formula: 1  
Value: 1

Cell: B17  
Formula: =\$B\$12\*\$A17  
Value: 20.56017

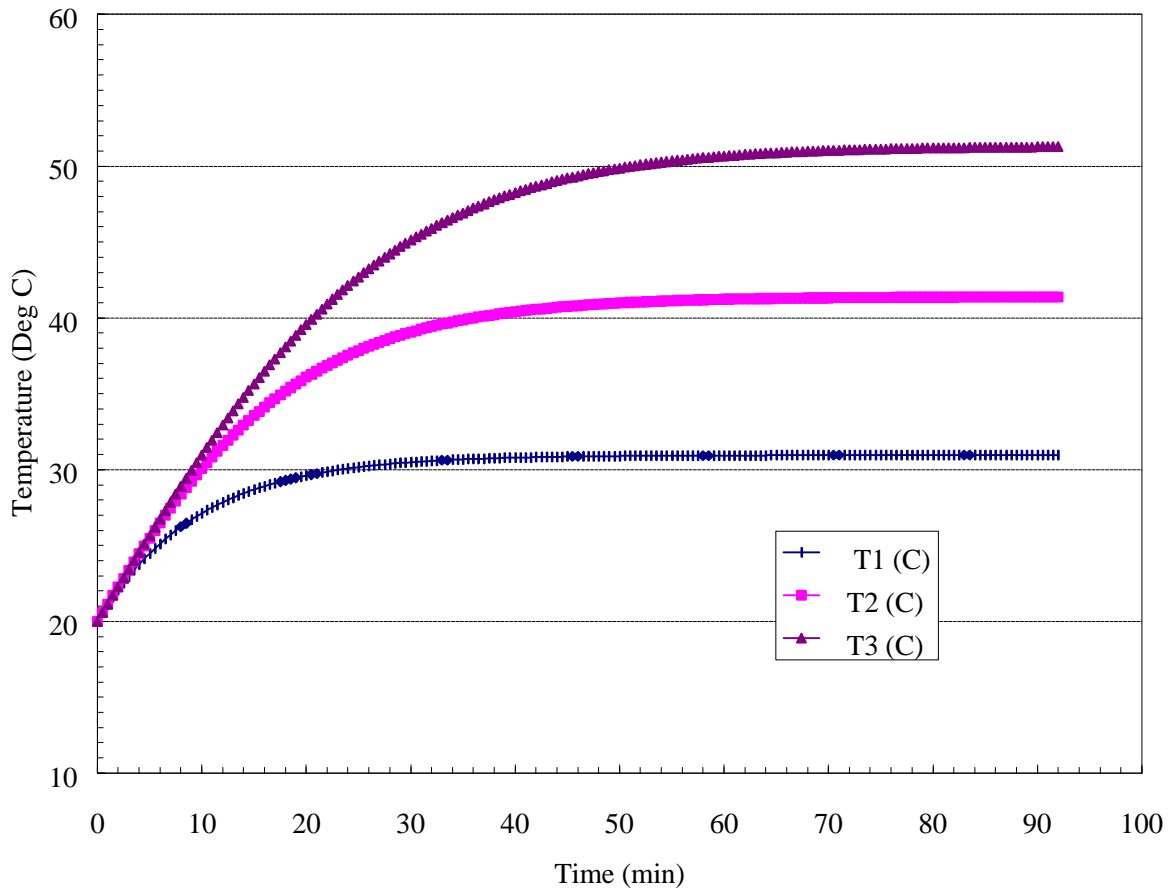
Cell: C17  
Formula: =rk4a(\$B\$12,\$B\$16,\$C16,\$D16,\$E16,0,0,\$B\$6)  
Value: 20.56017

Cell: D17  
Formula: =rk4a(\$B\$12,\$B\$16,\$C16,\$D16,\$E16,0,0,\$B\$6)  
Value: 20.57405

Cell: E17  
Formula: =rk4a(\$B\$12,\$B\$16,\$C16,\$D16,\$E16,0,0,\$B\$6)  
Value: 20.57428

EX Figure (6-1)

Dynamic Temperature Response in the Three Tanks



**Excel Problem 7 Solution****(a)**

The second order ODE can be broken into two first order equations as follows:

$$dC_A/dz = y \quad \text{EX Eq (7-1)}$$

$$dy/dz = k \cdot C_A / D_{AB} \quad \text{EX Eq (7-2)}$$

EX Spreadsheet (7-10) is set up to use the 4<sup>th</sup> order Runge-Kutta routine rk4 (EX Listing (7-1)). The right hand sides of the equations are given in EX Listing (7-2)

Since the initial value of y is not known Goal Seek is used to search for the initial value such that the y is zero at z = L. The thickness is arbitrarily divided into 50 increments. The search can be thought of as the outer loop and the integration as the inner loop.

**(b)**

User defined function ConA [EX Listing (7-3)] was written in VBA to evaluate the theoretical solution.

**EX Spreadsheet (7-1)**

Problem 7  
Diffusion With Chemical Reaction in a One Dimensional Slab

## Parameters

CA0 kg mol/m <sup>3</sup>	0.2
k - 1/s	0.001
DAB - m <sup>2</sup> /s	1.20E-09
L - m	0.001
h	0.00002

Index	Thickness - z	CA	y	CA - Theoretical Solution
0	0		0.2	0.2
1	0.00002	0.197394964	-128.5996599	0.197394964
2	0.00004	0.194855728	-125.330995	0.194855728
3	0.00006	0.192381446	-122.1041081	0.192381446
4	0.00008	0.189971292	-118.9179238	0.189971292
5	0.0001	0.187624464	-115.7713799	0.187624464
6	0.00012	0.18534018	-112.6634276	0.18534018
7	0.00014	0.183117677	-109.5930307	0.183117677
8	0.00016	0.180956215	-106.5591659	0.180956215
9	0.00018	0.178855073	-103.5608218	0.178855073
10	0.0002	0.176813552	-100.5969989	0.176813552
11	0.00022	0.17483097	-97.6667093	0.17483097
12	0.00024	0.172906667	-94.76897616	0.172906666
13	0.00026	0.17104	-91.90283355	0.17104
14	0.00028	0.169230349	-89.06732608	0.169230349

15	0.0003	0.167477109	-86.26150853	0.167477109
16	0.00032	0.165779697	-83.48444562	0.165779697
17	0.00034	0.164137546	-80.73521164	0.164137546
18	0.00036	0.162550109	-78.01289013	0.162550109
19	0.00038	0.161016857	-75.31657365	0.161016857
20	0.0004	0.159537279	-72.64536339	0.159537279
21	0.00042	0.158110881	-69.99836892	0.158110881
22	0.00044	0.156737188	-67.37470789	0.156737188
23	0.00046	0.155415743	-64.77350572	0.155415743
24	0.00048	0.154146104	-62.19389532	0.154146104
25	0.0005	0.152927849	-59.63501679	0.152927849
26	0.00052	0.151760571	-57.09601715	0.151760571
27	0.00054	0.150643881	-54.57605005	0.150643881
28	0.00056	0.149577408	-52.07427547	0.149577408
29	0.00058	0.148560795	-49.58985946	0.148560795
30	0.0006	0.147593703	-47.12197387	0.147593703
31	0.00062	0.146675811	-44.66979604	0.146675811
32	0.00064	0.145806812	-42.23250855	0.145806812
33	0.00066	0.144986417	-39.80929896	0.144986417
34	0.00068	0.144214352	-37.3993595	0.144214352
35	0.0007	0.143490359	-35.00188684	0.143490359
36	0.00072	0.142814198	-32.6160818	0.142814198
37	0.00074	0.142185644	-30.24114909	0.142185643
38	0.00076	0.141604485	-27.87629704	0.141604485
39	0.00078	0.14107053	-25.52073735	0.14107053
40	0.0008	0.140583599	-23.17368481	0.140583599
41	0.00082	0.140143531	-20.83435704	0.140143531
42	0.00084	0.139750178	-18.50197425	0.139750178
43	0.00086	0.139403411	-16.17575896	0.139403411
44	0.00088	0.139103112	-13.85493574	0.139103112
45	0.0009	0.138849183	-11.53873096	0.138849182
46	0.00092	0.138641537	-9.226372531	0.138641537
47	0.00094	0.138480107	-6.917089643	0.138480107
48	0.00096	0.138364838	-4.610112515	0.138364838
49	0.00098	0.138295692	-2.304672135	0.138295692
50	0.001	0.138272646	9.19265E-14	0.138272646

### EX Listing (7-1)

Public Function rk4(h, x, y1, y2, y3, y4, y5, nr, prm)

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'h = step size

'x = independent variable

'y1, y2, y3, y4, y5 = dependent variables

'nr = number of dependent variable to return =rk4

'prm a parameter vector of unspecified length

'kij : i is the k value, j is the equation number or dependent variable

$k11 = \text{fff1}(x, y1, y2, y3, y4, y5, \text{prm})$   
 $k12 = \text{fff2}(x, y1, y2, y3, y4, y5, \text{prm})$   
 $k13 = \text{fff3}(x, y1, y2, y3, y4, y5, \text{prm})$   
 $k14 = \text{fff4}(x, y1, y2, y3, y4, y5, \text{prm})$   
 $k15 = \text{fff5}(x, y1, y2, y3, y4, y5, \text{prm})$

$k21 = \text{fff1}(x + 0.5 * h, y1 + 0.5 * h * k11, y2 + 0.5 * h * k12, y3 + 0.5 * h * k13, y4 + 0.5 * h * k14, y5 + 0.5 * h * k15, \text{prm})$   
 $k22 = \text{fff2}(x + 0.5 * h, y1 + 0.5 * h * k11, y2 + 0.5 * h * k12, y3 + 0.5 * h * k13, y4 + 0.5 * h * k14, y5 + 0.5 * h * k15, \text{prm})$   
 $k23 = \text{fff3}(x + 0.5 * h, y1 + 0.5 * h * k11, y2 + 0.5 * h * k12, y3 + 0.5 * h * k13, y4 + 0.5 * h * k14, y5 + 0.5 * h * k15, \text{prm})$   
 $k24 = \text{fff4}(x + 0.5 * h, y1 + 0.5 * h * k11, y2 + 0.5 * h * k12, y3 + 0.5 * h * k13, y4 + 0.5 * h * k14, y5 + 0.5 * h * k15, \text{prm})$   
 $k25 = \text{fff5}(x + 0.5 * h, y1 + 0.5 * h * k11, y2 + 0.5 * h * k12, y3 + 0.5 * h * k13, y4 + 0.5 * h * k14, y5 + 0.5 * h * k15, \text{prm})$

$k31 = \text{fff1}(x + 0.5 * h, y1 + 0.5 * h * k21, y2 + 0.5 * h * k22, y3 + 0.5 * h * k23, y4 + 0.5 * h * k24, y5 + 0.5 * h * k25, \text{prm})$   
 $k32 = \text{fff2}(x + 0.5 * h, y1 + 0.5 * h * k21, y2 + 0.5 * h * k22, y3 + 0.5 * h * k23, y4 + 0.5 * h * k24, y5 + 0.5 * h * k25, \text{prm})$   
 $k33 = \text{fff3}(x + 0.5 * h, y1 + 0.5 * h * k21, y2 + 0.5 * h * k22, y3 + 0.5 * h * k23, y4 + 0.5 * h * k24, y5 + 0.5 * h * k25, \text{prm})$   
 $k34 = \text{fff4}(x + 0.5 * h, y1 + 0.5 * h * k21, y2 + 0.5 * h * k22, y3 + 0.5 * h * k23, y4 + 0.5 * h * k24, y5 + 0.5 * h * k25, \text{prm})$   
 $k35 = \text{fff5}(x + 0.5 * h, y1 + 0.5 * h * k21, y2 + 0.5 * h * k22, y3 + 0.5 * h * k23, y4 + 0.5 * h * k24, y5 + 0.5 * h * k25, \text{prm})$

$k41 = \text{fff1}(x + h, y1 + h * k31, y2 + h * k32, y3 + h * k33, y4 + h * k34, y5 + h * k35, \text{prm})$   
 $k42 = \text{fff2}(x + h, y1 + h * k31, y2 + h * k32, y3 + h * k33, y4 + h * k34, y5 + h * k35, \text{prm})$   
 $k43 = \text{fff3}(x + h, y1 + h * k31, y2 + h * k32, y3 + h * k33, y4 + h * k34, y5 + h * k35, \text{prm})$   
 $k44 = \text{fff4}(x + h, y1 + h * k31, y2 + h * k32, y3 + h * k33, y4 + h * k34, y5 + h * k35, \text{prm})$   
 $k45 = \text{fff5}(x + h, y1 + h * k31, y2 + h * k32, y3 + h * k33, y4 + h * k34, y5 + h * k35, \text{prm})$

Select Case nr

Case 1

$$\text{rk4} = y1 + (h / 6) * (k11 + 2 * k21 + 2 * k31 + k41)$$

Case 2

$$\text{rk4} = y2 + (h / 6) * (k12 + 2 * k22 + 2 * k32 + k42)$$

Case 3

$$\text{rk4} = y3 + (h / 6) * (k13 + 2 * k23 + 2 * k33 + k43)$$

Case 4

$$\text{rk4} = y4 + (h / 6) * (k14 + 2 * k24 + 2 * k34 + k44)$$

Case 5

$$\text{rk4} = y5 + (h / 6) * (k15 + 2 * k25 + 2 * k35 + k45)$$

End Select

End Function

**EX Listing (7-2)**

'These functions must be modified for actual problem usage

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'prm(1) = CAO

'prm(2) = k

'prm(3) = DAB

'prm(4) = L

'prm(5) = h

Public Function fff1(x, y1, y2, y3, y4, y5, prm)

fff1 = y2

End Function

Public Function fff2(x, y1, y2, y3, y4, y5, prm)

fff2 = prm(2) \* y1 / prm(3)

End Function

Public Function fff3(x, y1, y2, y3, y4, y5, prm)

fff3 = 0

End Function

Public Function fff4(x, y1, y2, y3, y4, y5, prm)

fff4 = 0

End Function

Public Function fff5(x, y1, y2, y3, y4, y5, prm)

fff5 = 0

End Function

**EX Listing (7-3)**

Public Function ConA(z, prm)

'prm(1) = CAO

'prm(2) = k

'prm(3) = DAB

'prm(4) = L

'prm(5) = h

t1 = Sqr(prm(2) / prm(3))

t2 = 1 - z / prm(4)

arg = prm(4) \* t1

```
Bot = Application.Cosh(arg)
Top = Application.Cosh(arg * t2)
```

```
ConA = prm(1) * Top / Bot
```

```
End Function
```

### Excel Problem 8 Solution

Two ordinary differential equations are solved for L (the moles of liquid remaining) and T (the bubble point temperature) with  $x_2$  (mole fraction of toluene) as the independent variable. Reference (2) discusses the approach of differentiating the bubble point equation to derive an express for  $dT/dx_2$ .

EX Listing (6-2) and EX Listing (8-1) are the user defined functions were used to carry out the integration. EX Spreadsheet (8-1) presents the results

### EX Spreadsheet (8-1)

Problem 8  
Binary Batch Distillation

Parameters	Value	Solution of initial bubble point Temperature Use Goal Seek	
A1 (Benzene)	6.90565		
B1 (Benzene)	1211.033	Initial x1	0.6
C1 (Benzene)	220.79	Initial x2	0.4
A2(Toluene)	6.95464		
B2 (Toluene)	1344.8	T(bpt)	95.56839
C2 (Toluene)	219.482	PA (mmHg)	1195.663
P (mmHg)	912	PB (mm Hg)	485.4187
$x_2$ (initial)	0.4	k1	1.311034
L (initial) moles	100	k2	0.532257
T(initial)	95.56839	Bpt Eq	-0.000477
h	0.01		

Index	$x_2$	L (moles)	T(bpt) - C
0	0.4	100	95.56839
1	0.41	94.83341	95.84216
2	0.42	90.00233	96.1181
3	0.43	85.4779	96.39625
4	0.44	81.2343	96.67663
5	0.45	77.2484	96.95927
6	0.46	73.49942	97.24421
7	0.47	69.96863	97.53147
8	0.48	66.63913	97.82108
9	0.49	63.49562	98.11308
10	0.5	60.52423	98.4075
11	0.51	57.71239	98.70437
12	0.52	55.04861	99.00373
13	0.53	52.52244	99.30562
14	0.54	50.12434	99.61006
15	0.55	47.84556	99.91709
16	0.56	45.6781	100.2267
17	0.57	43.61459	100.5391
18	0.58	41.64828	100.8541
19	0.59	39.77293	101.1719
20	0.6	37.98279	101.4924
21	0.61	36.27256	101.8158
22	0.62	34.63732	102.142
23	0.63	33.07253	102.4712

24	0.64	31.57396	102.8033
25	0.65	30.13769	103.1383
26	0.66	28.76008	103.4764
27	0.67	27.43771	103.8176
28	0.68	26.16743	104.1619
29	0.69	24.94626	104.5094
30	0.7	23.77144	104.8601
31	0.71	22.64036	105.214
32	0.72	21.55059	105.5713
33	0.73	20.49983	105.9319
34	0.74	19.48592	106.2959
35	0.75	18.50683	106.6634
36	0.76	17.56062	107.0345
37	0.77	16.64546	107.409
38	0.78	15.75961	107.7873
39	0.79	14.90142	108.1692
40	0.8	14.06929	108.5548

### EX Listing (8-1)

'These functions must be modified for actual problem usage

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'prm(1) = A1 for Benzene

'prm(2) = B1 for Benzene

'prm(3) = C1 for benzene

'prm(4) = A2 for Toluene

'prm(5) = B2 for Toluene

'prm(6) = C2 for Toluene

' prm(7) = Pressure mmHg

'prm(8) = x2 initial

'prm(9) = initial moles liquid

'prm(10) = Initial bpt

'y1 is L

'y2 is T(bubblepoint)

Public Function fff1(x, y1, y2, y3, y4, y5, prm)

Ptol = 10 ^ (prm(4) - prm(5) / (y2 + prm(6)))

k2 = Ptol / prm(7)

x2 = x

fff1 = y1 / (x2 \* (k2 - 1))

End Function

Public Function fff2(x, y1, y2, y3, y4, y5, prm)

Pben = 10 ^ (prm(1) - prm(2) / (y2 + prm(3)))

Ptol = 10 ^ (prm(4) - prm(5) / (y2 + prm(6)))

k1 = Pben / prm(7)

k2 = Ptol / prm(7)

```
t1 = k2 - k1
t2 = -prm(2) / (prm(3) + y2) ^ 2
t3 = -prm(5) / (prm(6) + y2) ^ 2
x2 = x
x1 = 1 - x2
t4 = x1 * k1 * t2
t5 = x2 * k2 * t3
fff2 = t1 / (Log(10) * (t4 + t5))
```

End Function

```
Public Function fff3(x, y1, y2, y3, y4, y5, prm)
```

```
fff3 = 0
```

End Function

```
Public Function fff4(x, y1, y2, y3, y4, y5, prm)
```

```
fff4 = 0
```

End Function

```
Public Function fff5(x, y1, y2, y3, y4, y5, prm)
```

```
fff5 = 0
```

End Function

## Excel Problem 9 Solution

(a)

The three differential equations given in Reference (1), Eq (30), Eq (36) and Eq (39) were integrated with the user defined function rk4 (EX Listing (7-1)). The right hand sides are shown in EX Listing (9-1).

A chart of the three independent variables X, T/1000 and y vs. W is given in EX Figure (9-1).

(b)

An explanation of the “knee” in EX Figure (9-1) is given in reference (2).

(c)

EX Figure (9-2) is a plot of CA and CC. User defined functions ConA and ConC (EX Listing (9-2)) were used to evaluate CA and CC as given in Reference (1) Eq (34) and Eq (35).

### EX Spreadsheet (9-1)

Problem 9

Reversible, Exothermic, Gas Phase Reaction in a Catalytic Reactor

Parameters	Value	Index	Temp Reduced P							
			W	Convrsn		T	y	T/1000	CA	CC
				X						
$C_{PA}$ - J/g-mol- K	40	0	0	0	450	1	0.45	0.271	0	
$C_{PC}$ -J/g-mol-K	80	1	0.4	0.00302	453.095	0.99698749	0.4530948	0.267933	0.000405	
Del $H_R$ - J/g-mol	-40,000	2	0.8	0.0062	456.354	0.99394929	0.4563537	0.264784	0.000826	
$E_A$ - J/g mol K	41,800	3	1.2	0.00958	459.793	0.99088435	0.4597931	0.261546	0.001264	
k - dm <sup>6</sup> /kg-min-mol @ 450 K	0.5	4	1.6	0.01315	463.432	0.98779154	0.4634318	0.258212	0.001721	
$K_C$ - dm <sup>3</sup> /g-mol @450 K)	25,000	5	2	0.01696	467.291	0.98466696	0.4672913	0.254774	0.002197	
$C_{AO}$ - g-mol/dm <sup>3</sup>	0.271	6	2.4	0.02101	471.397	0.98151716	0.4713968	0.251222	0.002696	
$T_o$ - K	450	7	2.8	0.02535	475.777	0.9783327	0.4757772	0.247544	0.003219	
R - J/g-mol-K	8.314	8	3.2	0.03001	480.467	0.97511457	0.4804667	0.24373	0.00377	
$F_{AO}$ g.-mol/min	5	9	3.6	0.03502	485.506	0.97186091	0.4855056	0.239763	0.00435	
$U_a$ - J/kg-min-K	0.8	10	4	0.04044	490.942	0.96856963	0.4909416	0.235628	0.004965	
$T_a$ - K	500	11	4.4	0.04632	496.832	0.96523841	0.4968323	0.231306	0.005617	
Alpha - kg <sup>-1</sup>	0.015	12	4.8	0.05273	503.247	0.96186463	0.5032469	0.226773	0.006312	
$P_o$ -atm	10	13	5.2	0.05976	510.27	0.95844528	0.5102703	0.222004	0.007056	
$y_{AO}$ - Pure A Feed	1	14	5.6	0.06752	518.007	0.95497694	0.518007	0.216966	0.007856	
Epsilon	-0.5	15	6	0.07614	526.588	0.95145566	0.5265876	0.211623	0.00872	
		16	6.4	0.08578	536.177	0.94787683	0.5361768	0.205928	0.009661	
Integration Parameter h	0.4	17	6.8	0.09665	546.986	0.94423505	0.5469858	0.199826	0.01069	
		18	7.2	0.10904	559.288	0.94052392	0.5592885	0.193251	0.011826	
		19	7.6	0.1233	573.445	0.93673579	0.5734454	0.186119	0.013089	
		20	8	0.13993	589.938	0.93286145	0.5899384	0.178331	0.014507	
		21	8.4	0.15957	609.42	0.92888966	0.6094196	0.169763	0.016116	

22	8.8	0.18312	632.779	0.9248067	0.632779	0.160266	0.017964
23	9.2	0.21181	661.232	0.9205958	0.661232	0.149673	0.020111
24	9.6	0.24726	696.403	0.91623678	0.6964028	0.137811	0.022635
25	10	0.29153	740.316	0.91170635	0.7403155	0.124556	0.025626
26	10.4	0.34667	795.029	0.90698043	0.795029	0.109952	0.029171
27	10.8	0.41353	861.37	0.90204036	0.8613696	0.09442	0.033288
28	11.2	0.48916	936.365	0.89688438	0.936365	0.078989	0.037819
29	11.6	0.56464	1011.09	0.89153817	1.011085	0.065231	0.042301
30	12	0.62802	1073.59	0.88605025	1.0735908	0.054576	0.046071
31	12.4	0.67229	1116.91	0.88046987	1.1169128	0.047456	0.048678
32	12.8	0.69886	1142.47	0.87482953	1.1424727	0.043224	0.050157
33	13.2	0.71327	1155.84	0.86914507	1.1558409	0.040869	0.050833
34	13.6	0.72064	1162.15	0.86342294	1.1621536	0.039568	0.051035
35	14	0.72428	1164.73	0.85766546	1.1647288	0.038817	0.050983
36	14.4	0.72602	1165.4	0.85187324	1.1654032	0.038342	0.0508
37	14.8	0.7268	1165.13	0.84604621	1.1651251	0.038003	0.05055
38	15.2	0.72711	1164.37	0.84018391	1.1643698	0.03773	0.050266
39	15.6	0.72718	1163.37	0.83428575	1.1633736	0.03749	0.049963
40	16	0.72712	1162.25	0.82835103	1.1622538	0.037266	0.049649
41	16.4	0.72699	1161.07	0.82237899	1.1610686	0.037049	0.049328
42	16.8	0.72683	1159.85	0.81636884	1.1598474	0.036834	0.049002
43	17.2	0.72664	1158.6	0.81031972	1.1586049	0.03662	0.048671
44	17.6	0.72643	1157.35	0.80423075	1.1573486	0.036406	0.048336
45	18	0.72622	1156.08	0.79810103	1.1560822	0.03619	0.047998
46	18.4	0.72599	1154.81	0.7919296	1.1548075	0.035973	0.047656
47	18.8	0.72576	1153.53	0.78571548	1.1535255	0.035755	0.047311
48	19.2	0.72551	1152.24	0.77945764	1.1522364	0.035534	0.046962
49	19.6	0.72526	1150.94	0.77315502	1.1509404	0.035312	0.046609
50	20	0.725	1149.64	0.7668065	1.1496374	0.035088	0.046252

## EX Listing (9-1)

These functions must be modified for actual problem usage

Written by EMRosen 6/19/97

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'prm(1) - CPA -J / g - mol - K

'prm(2) - CPC -J / g - mol - K

'prm(3)- Del HR - J / g - mol

'prm(4) - EA - J/g mol K

'prm(5) - k - dm6/kg-min-mol @ 450 K

'prm(6) - KC - dm3/g-mol @450 K)

'prm(7) - CAO -g - mol / dm3

'prm(8) - To - K

'prm(9) - R -J / g - mol - K

'prm(10) -FAO- g.-mol/min

'prm(11) -Ua -J / kg - Min - K

'prm(12) -Ta -K

'prm(13) -Alpha -kg - l

'prm(14) -Po -atm

'prm(15) - yAo - Pure A Feed

'prm(16) - Epsilon

'X - Conversion - y1  
'y - Reduced Pressure - y2  
'T - Temperature - y3  
'W - Catalyst Weight

Public Function fff1(W, y1, y2, y3, y4, y5, prm)

k = prm(5) \* Exp((prm(4) / prm(9)) \* (1 / prm(8) - 1 / y2))  
CA = ConA(W, y1, y2, y3, y4, y5, prm)  
CC = ConC(W, y1, y2, y3, y4, y5, prm)  
Kc = prm(6) \* Exp((prm(3) / prm(9)) \* (1 / prm(8) - 1 / y2))  
ra = -k \* (CA ^ 2 - CC / Kc)

fff1 = -ra / prm(10)

End Function

Public Function fff2(W, y1, y2, y3, y4, y5, prm)

k = prm(5) \* Exp((prm(4) / prm(9)) \* (1 / prm(8) - 1 / y2))  
CA = ConA(W, y1, y2, y3, y4, y5, prm)  
CC = ConC(W, y1, y2, y3, y4, y5, prm)  
Kc = prm(6) \* Exp((prm(3) / prm(9)) \* (1 / prm(8) - 1 / y2))  
ra = -k \* (CA ^ 2 - CC / Kc)

fff2 = (prm(11) \* (prm(12) - y2) + ra \* prm(3)) / (prm(10) \* prm(1))

End Function

Public Function fff3(W, y1, y2, y3, y4, y5, prm)

k = prm(5) \* Exp((prm(4) / prm(9)) \* (1 / prm(8) - 1 / y2))  
CA = ConA(W, y1, y2, y3, y4, y5, prm)  
CC = ConC(W, y1, y2, y3, y4, y5, prm)  
Kc = prm(6) \* Exp((prm(3) / prm(9)) \* (1 / prm(8) - 1 / y2))  
ra = -k \* (CA ^ 2 - CC / Kc)

fff3 = (-prm(13) \* (1 + prm(16) \* y1)) \* y2 / (2 \* y3 \* prm(8))

End Function

Public Function fff4(W, y1, y2, y3, y4, y5, prm)

fff4 = 0

End Function

Public Function fff5(W, y1, y2, y3, y4, y5, prm)

fff5 = 0

End Function

Public Function ConA(W, y1, y2, y3, y4, y5, prm)

$$t1 = (1 - y1) / (1 - 0.5 * y1)$$

$$\text{ConA} = \text{prm}(7) * t1 * y3 * \text{prm}(8) / y2$$

End Function

Public Function ConC(W, y1, y2, y3, y4, y5, prm)

$$\text{ConC} = 0.5 * \text{prm}(7) * y1 * y3 * \text{prm}(8) / ((1 - 0.5 * y1) * y2)$$

End Function

### EX Listing (9-2)

Public Function ConA(W, y1, y2, y3, y4, y5, prm)

$$t1 = (1 - y1) / (1 - 0.5 * y1)$$

$$\text{ConA} = \text{prm}(7) * t1 * y3 * \text{prm}(8) / y2$$

End Function

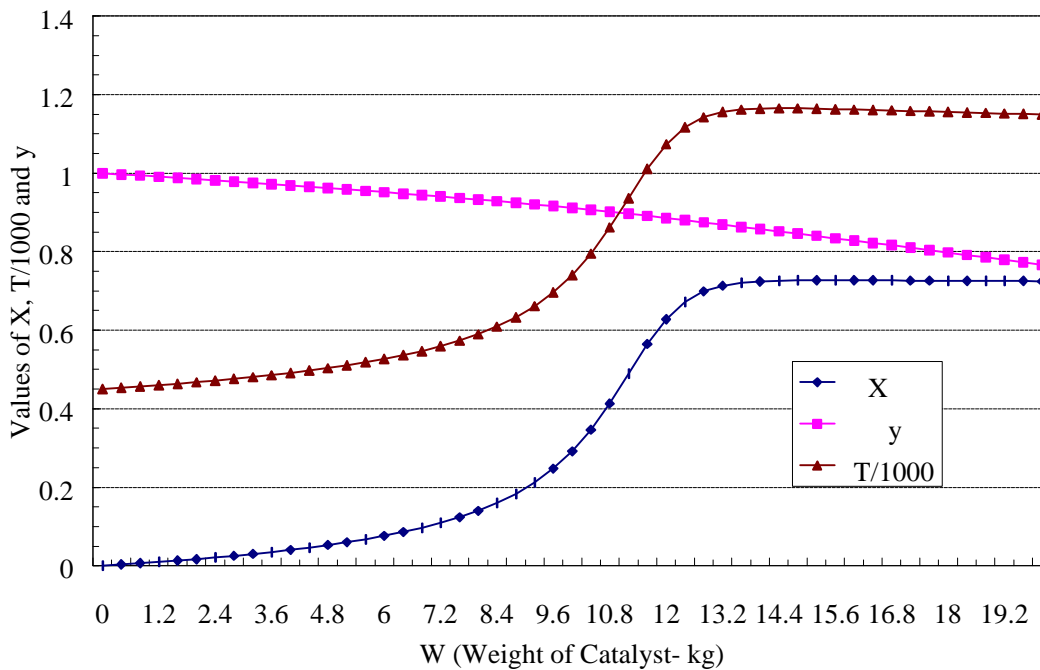
Public Function ConC(W, y1, y2, y3, y4, y5, prm)

$$\text{ConC} = 0.5 * \text{prm}(7) * y1 * y3 * \text{prm}(8) / ((1 - 0.5 * y1) * y2)$$

End Function

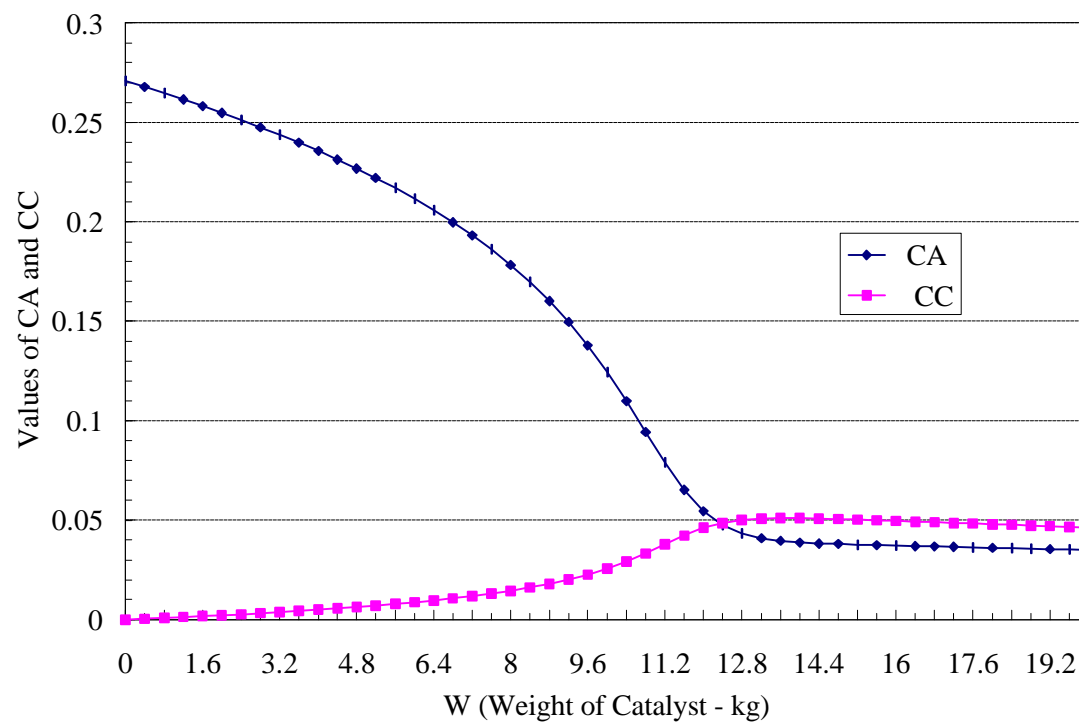
### EX Figure (9-1)

Reversible, Exothermic, Gas Phase Reaction in a Catalytic Reactor



EX Figure (9-2)

Reversible, Exothermic, Gas Phase Reaction in a Catalytic Reactor



### Excel Problem 10 Solution

All parts of this problem (a-e) were solved using the rk4 user defined function [EX Listing (7-1)]. The right hand side user defined functions fff1 to fff2 were modified to account for the changes in control and limits on  $q$ . Changes in  $K_c$  were made on the spreadsheet and passed to the user defined functions fff1 to fff2.

(a)

The spreadsheet resulting from PI control with  $K_c = 0$  (open loop) is given by EX Spreadsheet (10-1). The user defined functions for fff1...fff2 are given by EX Listing (10-1). The plot is given in EX Figure (10-1).

### EX Spreadsheet (10-1)

Problem 10(a)  
Dynamics of a Heated Tank with PI Temperature Control

Parameters	Value	Index	<-----Integration----->				
			Time (min)	T	To	Tm	errsum
rhoVCp - kJ/C	4000	0	0	80	80	80	0
Tis - C	60	1	0.5	80	80	80	0
taud - min	1	2	1	80	80	80	0
Kc - kJ/min-C	0	3	1.5	80	80	80	0
WCp - kJ/min-C	500	4	2	80	80	80	0
Tr - C	80	5	2.5	80	80	80	0
taum - min	5	6	3	80	80	80	0
taul - min	2	7	3.5	80	80	80	0
Integration Increment - h	0.5	8	4	80	80	80	0
		9	4.5	80	80	80	0
		10	5	80	80	80	0
		11	5.5	80	80	80	0
		12	6	80	80	80	0
		13	6.5	80	80	80	0
		14	7	80	80	80	0
		15	7.5	80	80	80	0
		16	8	80	80	80	0
		17	8.5	80	80	80	0
		18	9	80	80	80	0
		19	9.5	80	80	80	0
		20	10	79.79167	80.20833	80	0
		21	10.5	78.59255	80.23822	80.03406	-0.007257
		22	11	77.46608	79.4824	80.02134	-0.02373
		23	11.5	76.40787	78.4784	79.92278	-0.01336
		24	12	75.41377	77.42499	79.73452	0.068771
25	12.5	74.47989	76.39407	79.46461	0.265836		
26	13	73.6026	75.4101	79.12441	0.615901		
27	13.5	72.77846	74.47994	78.72553	1.151199		
28	14	72.00425	73.60395	78.27871	1.898347		
29	14.5	71.27695	72.78022	77.7936	2.878856		
30	15	70.59372	72.00609	77.27868	4.109703		
31	15.5	69.95188	71.27875	76.74138	5.603892		
32	16	69.34892	70.59543	76.18815	7.370964		
33	16.5	68.7825	69.95349	75.62456	9.417458		

34	17	68.2504	69.35045	75.05541	11.74732
35	17.5	67.75053	68.78393	74.48483	14.36227
36	18	67.28095	68.25174	73.91631	17.26214
37	18.5	66.83982	67.75179	73.35282	20.44512
38	19	66.42541	67.28214	72.79686	23.90806
39	19.5	66.03612	66.84093	72.2505	27.64665
40	20	65.67041	66.42646	71.71546	31.65566
41	20.5	65.32686	66.0371	71.19312	35.92907
42	21	65.00412	65.67133	70.68458	40.46024
43	21.5	64.70093	65.32772	70.19072	45.24204
44	22	64.41612	65.00493	69.71217	50.26697
45	22.5	64.14856	64.7017	69.24939	55.52725
46	23	63.89721	64.41684	68.8027	61.0149
47	23.5	63.66109	64.14924	68.37226	66.72183
48	24	63.43928	63.89785	67.9581	72.63992
49	24.5	63.2309	63.66169	67.56017	78.76103
50	25	63.03515	63.43984	67.17834	85.07707
51	25.5	62.85126	63.23143	66.81239	91.58004
52	26	62.67851	63.03565	66.46205	98.26208
53	26.5	62.51623	62.85173	66.12699	105.1154
54	27	62.36378	62.67895	65.80687	112.1326
55	27.5	62.22056	62.51664	65.50127	119.3062
56	28	62.08603	62.36416	65.2098	126.629
57	28.5	61.95964	62.22093	64.932	134.0941
58	29	61.84091	62.08637	64.66743	141.6948
59	29.5	61.72938	61.95996	64.41563	149.4245
60	30	61.6246	61.84121	64.17614	157.2771
61	30.5	61.52617	61.72966	63.94849	165.2464
62	31	61.4337	61.62486	63.73222	173.3267
63	31.5	61.34684	61.52642	63.52687	181.5124
64	32	61.26524	61.43394	63.33198	189.7981
65	32.5	61.18858	61.34706	63.14711	198.1787
66	33	61.11657	61.26545	62.97183	206.6494
67	33.5	61.04892	61.18878	62.8057	215.2054
68	34	60.98537	61.11675	62.64831	223.8422
69	34.5	60.92567	61.04909	62.49925	232.5557
70	35	60.86959	60.98553	62.35815	241.3416
71	35.5	60.8169	60.92582	62.22461	250.1963
72	36	60.76741	60.86973	62.09827	259.1158
73	36.5	60.72091	60.81703	61.97878	268.0968
74	37	60.67723	60.76753	61.86581	277.136
75	37.5	60.6362	60.72103	61.75902	286.23
76	38	60.59766	60.67734	61.65811	295.376
77	38.5	60.56145	60.63631	61.56277	304.571
78	39	60.52743	60.59775	61.47272	313.8123
79	39.5	60.49547	60.56154	61.38769	323.0974
80	40	60.46546	60.52752	61.30741	332.4238
81	40.5	60.43726	60.49556	61.23163	341.7893
82	41	60.41076	60.46553	61.16011	351.1915
83	41.5	60.38588	60.43733	61.09264	360.6285
84	42	60.3625	60.41083	61.02898	370.0982
85	42.5	60.34053	60.38594	60.96894	379.5989
86	43	60.3199	60.36256	60.91232	389.1287
87	43.5	60.30052	60.34059	60.85893	398.686
88	44	60.28231	60.31995	60.80859	408.2693
89	44.5	60.26521	60.30057	60.76115	417.877
90	45	60.24914	60.28236	60.71643	427.5077
91	45.5	60.23405	60.26525	60.67428	437.1601
92	46	60.21987	60.24918	60.63457	446.833
93	46.5	60.20654	60.23408	60.59716	456.5251
94	47	60.19403	60.2199	60.56192	466.2355

95	47.5	60.18228	60.20658	60.52872	475.9629
96	48	60.17123	60.19406	60.49745	485.7064
97	48.5	60.16086	60.1823	60.468	495.4651
98	49	60.15111	60.17126	60.44028	505.2381
99	49.5	60.14196	60.16088	60.41417	515.0246
100	50	60.13336	60.15114	60.38959	524.8237
101	50.5	60.12528	60.14198	60.36645	534.6348
102	51	60.11769	60.13338	60.34467	544.457
103	51.5	60.11056	60.1253	60.32417	554.2899
104	52	60.10386	60.1177	60.30487	564.1327
105	52.5	60.09756	60.11057	60.28671	573.9848
106	53	60.09165	60.10387	60.26962	583.8458
107	53.5	60.0861	60.09758	60.25354	593.715
108	54	60.08088	60.09167	60.23841	603.5921
109	54.5	60.07598	60.08611	60.22418	613.4765
110	55	60.07138	60.0809	60.21078	623.3678
111	55.5	60.06706	60.076	60.19818	633.2655
112	56	60.06299	60.07139	60.18633	643.1694
113	56.5	60.05918	60.06707	60.17518	653.0791
114	57	60.05559	60.063	60.16469	662.9942
115	57.5	60.05222	60.05919	60.15483	672.9143
116	58	60.04906	60.0556	60.14555	682.8392
117	58.5	60.04609	60.05223	60.13683	692.7687
118	59	60.04329	60.04907	60.12862	702.7023
119	59.5	60.04067	60.04609	60.12091	712.64
120	60	60.03821	60.0433	60.11365	722.5813
121	60.5	60.03589	60.04068	60.10683	732.5262
122	61	60.03372	60.03821	60.10041	742.4744
123	61.5	60.03167	60.0359	60.09438	752.4258
124	62	60.02976	60.03372	60.08871	762.38
125	62.5	60.02795	60.03168	60.08338	772.337
126	63	60.02626	60.02976	60.07836	782.2966
127	63.5	60.02467	60.02796	60.07365	792.2586
128	64	60.02317	60.02626	60.06922	802.2229
129	64.5	60.02177	60.02467	60.06505	812.1893
130	65	60.02045	60.02318	60.06114	822.1578
131	65.5	60.01921	60.02177	60.05746	832.1281
132	66	60.01805	60.02045	60.054	842.1003
133	66.5	60.01695	60.01921	60.05074	852.0741
134	67	60.01593	60.01805	60.04769	862.0495
135	67.5	60.01496	60.01696	60.04481	872.0264
136	68	60.01406	60.01593	60.04211	882.0047
137	68.5	60.0132	60.01496	60.03957	891.9842
138	69	60.0124	60.01406	60.03719	901.9651
139	69.5	60.01165	60.01321	60.03494	911.947
140	70	60.01095	60.01241	60.03284	921.9301
141	70.5	60.01028	60.01165	60.03086	931.9142
142	71	60.00966	60.01095	60.02899	941.8992
143	71.5	60.00907	60.01028	60.02724	951.8852
144	72	60.00853	60.00966	60.0256	961.872
145	72.5	60.00801	60.00908	60.02405	971.8596
146	73	60.00752	60.00853	60.0226	981.8479
147	73.5	60.00707	60.00801	60.02124	991.8369
148	74	60.00664	60.00752	60.01996	1001.827
149	74.5	60.00624	60.00707	60.01875	1011.817
150	75	60.00586	60.00664	60.01762	1021.808
151	75.5	60.0055	60.00624	60.01655	1031.799
152	76	60.00517	60.00586	60.01555	1041.791
153	76.5	60.00486	60.00551	60.01461	1051.784
154	77	60.00456	60.00517	60.01373	1061.777
155	77.5	60.00429	60.00486	60.0129	1071.77

156	78	60.00403	60.00456	60.01212	1081.764
157	78.5	60.00378	60.00429	60.01139	1091.758
158	79	60.00355	60.00403	60.0107	1101.752
159	79.5	60.00334	60.00378	60.01005	1111.747
160	80	60.00314	60.00355	60.00945	1121.742
161	80.5	60.00295	60.00334	60.00887	1131.738
162	81	60.00277	60.00314	60.00834	1141.733
163	81.5	60.0026	60.00295	60.00783	1151.729
164	82	60.00244	60.00277	60.00736	1161.726
165	82.5	60.00229	60.0026	60.00691	1171.722
166	83	60.00216	60.00244	60.0065	1181.719
167	83.5	60.00202	60.00229	60.0061	1191.716
168	84	60.0019	60.00216	60.00573	1201.713
169	84.5	60.00179	60.00203	60.00539	1211.71
170	85	60.00168	60.0019	60.00506	1221.707
171	85.5	60.00158	60.00179	60.00475	1231.705
172	86	60.00148	60.00168	60.00447	1241.702
173	86.5	60.00139	60.00158	60.0042	1251.7
174	87	60.00131	60.00148	60.00394	1261.698
175	87.5	60.00123	60.00139	60.0037	1271.696
176	88	60.00115	60.00131	60.00348	1281.695
177	88.5	60.00108	60.00123	60.00327	1291.693
178	89	60.00102	60.00115	60.00307	1301.691
179	89.5	60.00096	60.00108	60.00289	1311.69
180	90	60.0009	60.00102	60.00271	1321.688
181	90.5	60.00084	60.00096	60.00255	1331.687
182	91	60.00079	60.0009	60.00239	1341.686
183	91.5	60.00074	60.00084	60.00225	1351.685
184	92	60.0007	60.00079	60.00211	1361.684
185	92.5	60.00066	60.00075	60.00198	1371.683
186	93	60.00062	60.0007	60.00186	1381.682
187	93.5	60.00058	60.00066	60.00175	1391.681
188	94	60.00054	60.00062	60.00165	1401.68
189	94.5	60.00051	60.00058	60.00155	1411.679
190	95	60.00048	60.00055	60.00145	1421.678
191	95.5	60.00045	60.00051	60.00136	1431.678
192	96	60.00042	60.00048	60.00128	1441.677
193	96.5	60.0004	60.00045	60.0012	1451.676
194	97	60.00037	60.00042	60.00113	1461.676
195	97.5	60.00035	60.0004	60.00106	1471.675
196	98	60.00033	60.00037	60.001	1481.675
197	98.5	60.00031	60.00035	60.00094	1491.674
198	99	60.00029	60.00033	60.00088	1501.674
199	99.5	60.00027	60.00031	60.00083	1511.673
200	100	60.00026	60.00029	60.00078	1521.673
201	100.5	60.00024	60.00027	60.00073	1531.673
202	101	60.00023	60.00026	60.00069	1541.672
203	101.5	60.00021	60.00024	60.00064	1551.672
204	102	60.0002	60.00023	60.00061	1561.672

**EX Listing (10-1)**

'These functions must be modified for actual problem usage

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## Proportional and Integral Controllers

```
'prm(1) -rhoVCp -kJ / C
'prm(2) -Tis -C
'prm(3) -taud -Min
'prm(4) -Kc -b kJ/min-C
'prm(5) -WCp -kJ / Min - C
'prm(6) -Tr -C
'prm(7) -taum -Min
'prm(8) -tauI -Min
```

```
'x - time
'y1 - T
'y2 - To
'y3 - Tm
'y4 - errsum
```

Public Function fff1(x, y1, y2, y3, y4, y5, prm)

```
step = 1
If x < 10 Then
    step = 0
End If
```

```
ti = prm(2) + step * (-20)
```

```
qs = prm(5) * (prm(6) - prm(2))
q = qs + prm(4) * (prm(6) - y3) + prm(4) * y4 / prm(8)
dTdt = (prm(5) * (ti - y1) + q) / prm(1)
```

```
fff1 = dTdt
End Function
```

Public Function fff2(x, y1, y2, y3, y4, y5, prm)

```
step = 1
If x < 10 Then
    step = 0
End If
```

```
ti = prm(2) + step * (-20)
```

```
qs = prm(5) * (prm(6) - prm(2))
q = qs + prm(4) * (prm(6) - y3) + prm(4) * y4 / prm(8)
dTdt = (prm(5) * (ti - y1) + q) / prm(1)
```

```
fff2 = (y1 - y2 - (prm(3) / 2) * dTdt) * 2 / prm(3)
```

```
End Function
```

Public Function fff3(x, y1, y2, y3, y4, y5, prm)

fff3 = (y2 - y3) / prm(7)

End Function

Public Function fff4(x, y1, y2, y3, y4, y5, prm)

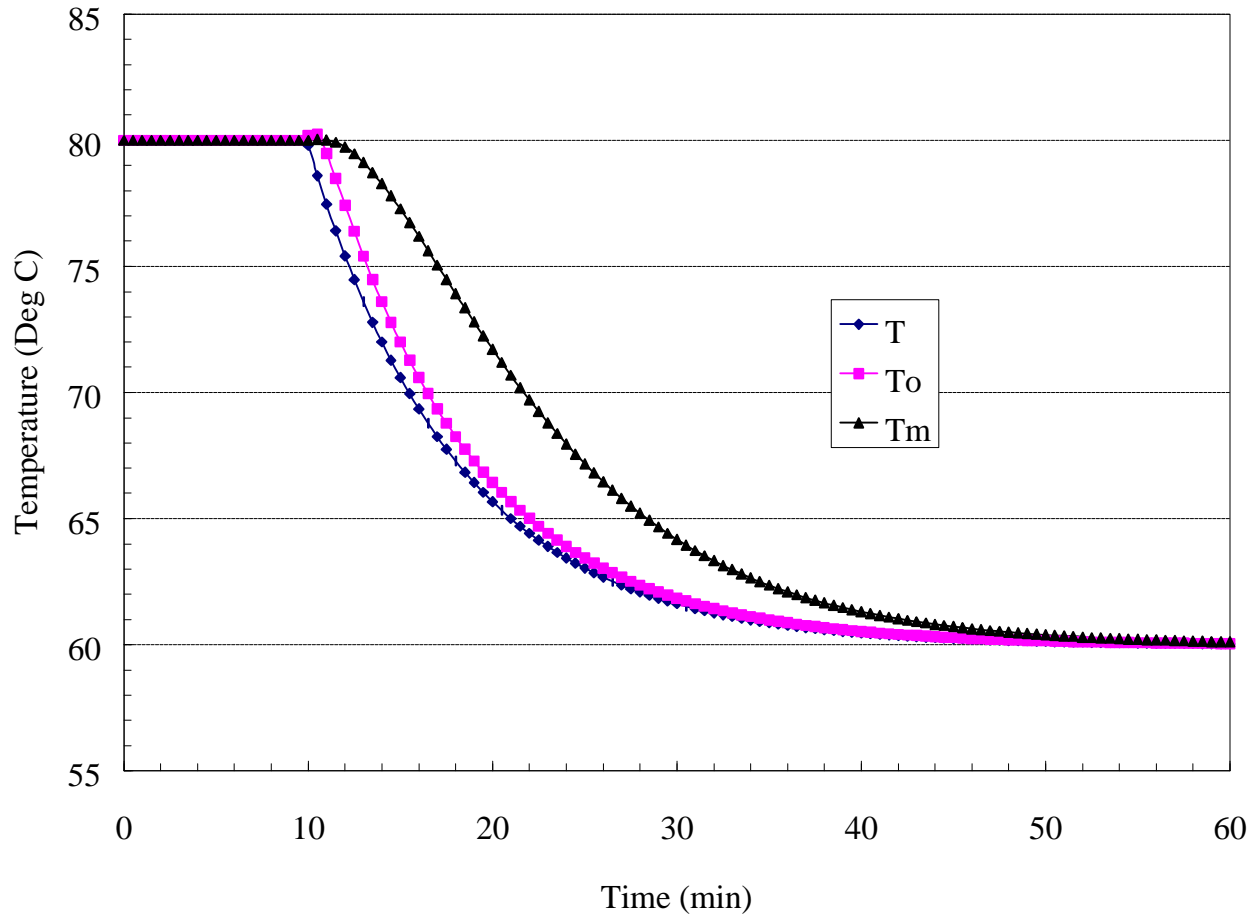
fff4 = prm(6) - y3

End Function

Public Function fff5(x, y1, y2, y3, y4, y5, prm)

fff5 = 0

End Function

**EX Figure (10-1)****Dynamics of a Heated Tank - PI Control -  $K_c = 0$** **(b)**

The spreadsheet resulting from PI control with  $K_c = 50$  is given by EX Spreadsheet (10-2). The user defined functions for fff1...fff2 are given by EX Listing (10-2). The plot is given in EX Figure (10-2).

**EX Spreadsheet (10-2)**

Problem 10(b)  
Dynamics of a Heated Tank with PI Temperature Control

Parameters	Value	Index	Time (min)	<-----Integration----->			
				T	To	Tm	errsum
rhoVCp - kJ/C	4000	0	0	80	80	80	0
Tis - C	60	1	0.5	80	80	80	0

taud - min	1	2	1	80	80	80	0
Kc - kJ/min-C	50	3	1.5	80	80	80	0
WCp - kJ/min-C	500	4	2	80	80	80	0
Tr - C	80	5	2.5	80	80	80	0
taum - min	5	6	3	80	80	80	0
taul - min	2	7	3.5	80	80	80	0
		8	4	80	80	80	0
Integration Increment - h	0.5	9	4.5	80	80	80	0
		10	5	80	80	80	0
		11	5.5	80	80	80	0
		12	6	80	80	80	0
		13	6.5	80	80	80	0
		14	7	80	80	80	0
		15	7.5	80	80	80	0
		16	8	80	80	80	0
		17	8.5	80	80	80	0
		18	9	80	80	80	0
		19	9.5	80	80	80	0
		20	10	79.79167	80.20833	80	0
		21	10.5	78.59245	80.23825	80.03406	-0.00726
		22	11	77.46574	79.4824	80.02134	-0.02373
		23	11.5	76.40761	78.47809	79.92277	-0.01336
		24	12	75.41458	77.42429	79.73446	0.068781
		25	12.5	74.48353	76.39336	79.46449	0.265892
		26	13	73.61157	75.41043	79.12428	0.616027
		27	13.5	72.79602	74.48309	78.72555	1.151365
		28	14	72.03439	73.61244	78.27927	1.898389
		29	14.5	71.32435	72.79732	77.7953	2.878365
		30	15	70.66369	72.03579	77.28242	4.107896
		31	15.5	70.05036	71.32572	76.74839	5.599458
		32	16	69.4824	70.66502	76.20001	7.361888
		33	16.5	68.95797	70.05163	75.64324	9.400837
		34	17	68.4753	69.48361	75.0833	11.71916
		35	17.5	68.03272	68.95912	74.52471	14.3173
		36	18	67.62861	68.4764	73.9714	17.19357
		37	18.5	67.26146	68.03376	73.42675	20.34445
		38	19	66.92977	67.62962	72.89369	23.76488
		39	19.5	66.63213	67.26242	72.3747	27.44841
		40	20	66.36717	66.9307	71.87188	31.38748
		41	20.5	66.13357	66.63303	71.387	35.57354
		42	21	65.93005	66.36804	70.92153	39.99724
		43	21.5	65.75537	66.13441	70.47666	44.64857
		44	22	65.60833	65.93087	70.05336	49.51698
		45	22.5	65.48776	65.75616	69.65238	54.59149
		46	23	65.39253	65.6091	69.27428	59.86079
		47	23.5	65.32154	65.48851	68.91945	65.31333
		48	24	65.2737	65.39326	68.58815	70.93741
		49	24.5	65.24799	65.32224	68.28049	76.72124
		50	25	65.24337	65.27439	67.99648	82.65298
		51	25.5	65.25886	65.24866	67.73602	88.72083
		52	26	65.2935	65.24402	67.49893	94.91306
		53	26.5	65.34634	65.2595	67.28494	101.218
		54	27	65.41646	65.29412	67.09372	107.6243
		55	27.5	65.50297	65.34694	66.92488	114.1206
		56	28	65.60499	65.41705	66.77798	120.6958
		57	28.5	65.72169	65.50354	66.65253	127.339
		58	29	65.85222	65.60555	66.548	134.0398
		59	29.5	65.99579	65.72223	66.46385	140.7876
		60	30	66.15161	65.85275	66.39948	147.5726
		61	30.5	66.31892	65.99631	66.3543	154.385
		62	31	66.49698	66.15212	66.32767	161.2152

63	31.5	66.68506	66.31941	66.31897	168.0543
64	32	66.88247	66.49746	66.32754	174.8934
65	32.5	67.08852	66.68552	66.35273	181.724
66	33	67.30255	66.88292	66.39388	188.538
67	33.5	67.52393	67.08896	66.45032	195.3276
68	34	67.75203	67.30298	66.52139	202.0852
69	34.5	67.98625	67.52434	66.60642	208.8039
70	35	68.226	67.75242	66.70475	215.4766
71	35.5	68.47073	67.98663	66.81573	222.097
72	36	68.71989	68.22637	66.93871	228.6589
73	36.5	68.97296	68.47109	67.07305	235.1564
74	37	69.22942	68.72023	67.21811	241.584
75	37.5	69.48879	68.97329	67.37327	247.9366
76	38	69.7506	69.22974	67.53791	254.2092
77	38.5	70.0144	69.4891	67.71144	260.3972
78	39	70.27975	69.7509	67.89325	266.4964
79	39.5	70.54623	70.01469	68.08278	272.5027
80	40	70.81345	70.28002	68.27945	278.4124
81	40.5	71.08101	70.5465	68.48271	284.2221
82	41	71.34856	70.8137	68.69203	289.9287
83	41.5	71.61573	71.08125	68.90687	295.5292
84	42	71.8822	71.34879	69.12673	301.021
85	42.5	72.14765	71.61595	69.35112	306.4017
86	43	72.41177	71.88241	69.57954	311.6692
87	43.5	72.67427	72.14785	69.81154	316.8216
88	44	72.93489	72.41196	70.04666	321.8571
89	44.5	73.19335	72.67445	70.28446	326.7745
90	45	73.44942	72.93506	70.52452	331.5723
91	45.5	73.70287	73.19351	70.76643	336.2496
92	46	73.95347	73.44957	71.0098	340.8056
93	46.5	74.20103	73.70301	71.25426	345.2396
94	47	74.44535	73.95361	71.49944	349.5512
95	47.5	74.68626	74.20116	71.74498	353.7401
96	48	74.92358	74.44547	71.99056	357.8063
97	48.5	75.15717	74.68636	72.23586	361.7496
98	49	75.38688	74.92368	72.48056	365.5705
99	49.5	75.61258	75.15726	72.72439	369.2692
100	50	75.83414	75.38696	72.96705	372.8463
101	50.5	76.05147	75.61265	73.20828	376.3024
102	51	76.26447	75.83421	73.44784	379.6383
103	51.5	76.47303	76.05154	73.68548	382.8549
104	52	76.67709	76.26452	73.92098	385.9532
105	52.5	76.87657	76.47308	74.15412	388.9343
106	53	77.07141	76.67713	74.3847	391.7995
107	53.5	77.26156	76.8766	74.61254	394.5501
108	54	77.44698	77.07144	74.83746	397.1874
109	54.5	77.62763	77.26159	75.05929	399.7131
110	55	77.80349	77.447	75.27787	402.1287
111	55.5	77.97452	77.62765	75.49307	404.4358
112	56	78.14073	77.8035	75.70474	406.6362
113	56.5	78.3021	77.97453	75.91278	408.7317
114	57	78.45863	78.14073	76.11706	410.7241
115	57.5	78.61033	78.30209	76.31748	412.6153
116	58	78.75722	78.45862	76.51395	414.4072
117	58.5	78.89931	78.61032	76.70639	416.102
118	59	79.03662	78.75721	76.89472	417.7015
119	59.5	79.16919	78.89929	77.07887	419.208
120	60	79.29705	79.0366	77.25878	420.6234

**EX Listing (10-2)**

'These functions must be modified for actual problem usage

'Written by EMRosen 6/19/97

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'Proportional and Integral Controllers

'prm(1) -rhoVCp -kJ / C

'prm(2) -Tis -C

'prm(3) -taud -Min

'prm(4) -Kc -b kJ/min-C

'prm(5) -WCp -kJ / Min - C

'prm(6) -Tr -C

'prm(7) -taum -Min

'prm(8) -tauI -Min

'x - time

'y1 -T

'y2 - To

'y3 - Tm

'y4 - errsum

Public Function fff1(x, y1, y2, y3, y4, y5, prm)

step = 1

If x < 10 Then

step = 0

End If

ti = prm(2) + step \* (-20)

qs = prm(5) \* (prm(6) - prm(2))

q = qs + prm(4) \* (prm(6) - y3) + prm(4) \* y4 / prm(8)

dTdt = (prm(5) \* (ti - y1) + q) / prm(1)

fff1 = dTdt

End Function

Public Function fff2(x, y1, y2, y3, y4, y5, prm)

step = 1

If x < 10 Then

step = 0

End If

ti = prm(2) + step \* (-20)

```
qs = prm(5) * (prm(6) - prm(2))  
q = qs + prm(4) * (prm(6) - y3) + prm(4) * y4 / prm(8)  
dTdt = (prm(5) * (ti - y1) + q) / prm(1)
```

```
fff2 = (y1 - y2 - (prm(3) / 2) * dTdt) * 2 / prm(3)
```

```
End Function
```

```
Public Function fff3(x, y1, y2, y3, y4, y5, prm)
```

```
fff3 = (y2 - y3) / prm(7)
```

```
End Function
```

```
Public Function fff4(x, y1, y2, y3, y4, y5, prm)
```

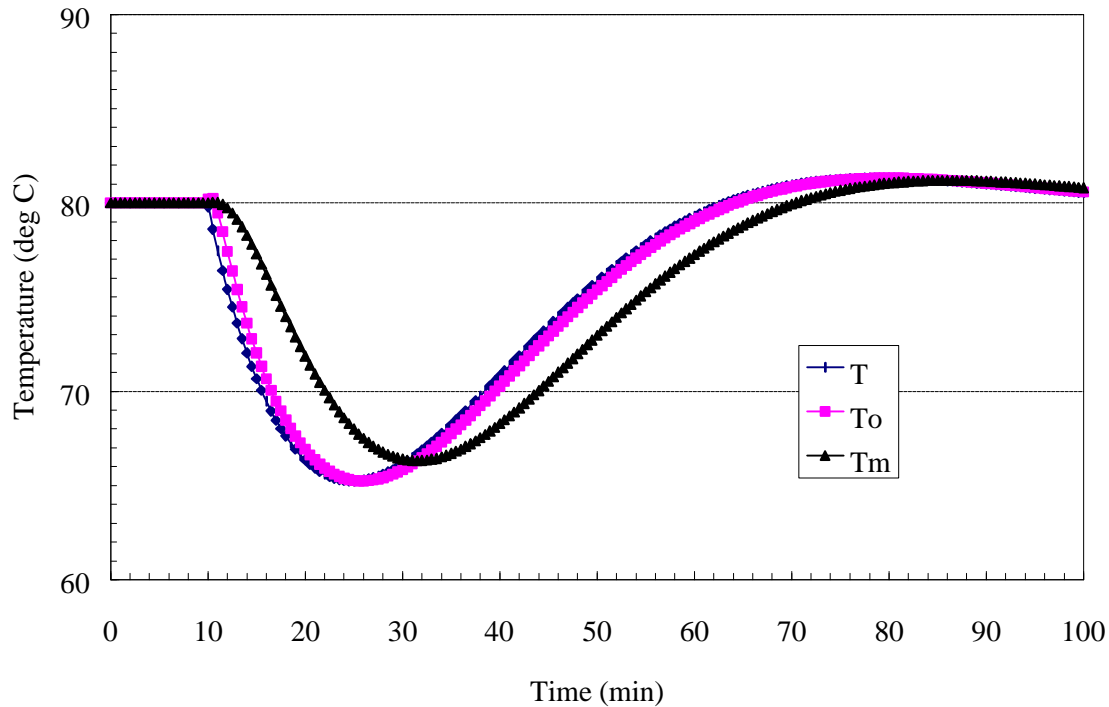
```
fff4 = prm(6) - y3
```

```
End Function
```

```
Public Function fff5(x, y1, y2, y3, y4, y5, prm)
```

```
fff5 = 0
```

```
End Function
```

**EX Figure (10-2)**Dynamics of a Heated Tank - PI Control for  $K_c = 0$ 

(c)

The spreadsheet resulting from PI control with  $K_c = 500$  is given by EX Spreadsheet (10-3). The user defined functions for  $fff1 \dots fff2$  are given by EX Listing (10-3). The plot is given in EX Figure (10-3).

**EX Spreadsheet (10-3)**

Problem 10(c)  
Dynamics of a Heated Tank with PI Temperature Control

Parameters	Value	Index	<-----Integration----->				
			Time (min)	T	To	Tm	errsum
rhoVCp - kJ/C	4000	0	0	80	80	80	0
Tis - C	60	1	0.5	80	80	80	0
taud - min	1	2	1	80	80	80	0
Kc - kJ/min-C	500	3	1.5	80	80	80	0
WCp - kJ/min-C	500	4	2	80	80	80	0
Tr - C	80	5	2.5	80	80	80	0
taum - min	5	6	3	80	80	80	0
taul - min	2	7	3.5	80	80	80	0
		8	4	80	80	80	0
Integration Increment - h	0.5	9	4.5	80	80	80	0
		10	5	80	80	80	0

11	5.5	80	80	80	0
12	6	80	80	80	0
13	6.5	80	80	80	0
14	7	80	80	80	0
15	7.5	80	80	80	0
16	8	80	80	80	0
17	8.5	80	80	80	0
18	9	80	80	80	0
19	9.5	80	80	80	0
20	10	79.79167	80.20833	80	0
21	10.5	78.59156	80.23845	80.03409	-0.00726
22	11	77.46267	79.4824	80.0214	-0.02376
23	11.5	76.40525	78.47535	79.92271	-0.0134
24	12	75.42196	77.41794	79.73396	0.068876
25	12.5	74.51637	76.38696	79.46337	0.266394
26	13	73.69249	75.41343	79.12301	0.617165
27	13.5	72.95434	74.5115	78.72577	1.152866
28	14	72.30578	73.68907	78.28429	1.89876
29	14.5	71.75037	72.95159	77.81063	2.873926
30	15	71.29123	72.30347	77.31616	4.091585
31	15.5	70.93094	71.74846	76.81154	5.559447
32	16	70.67143	71.28973	76.30676	7.280063
33	16.5	70.51396	70.92988	75.81113	9.251155
34	17	70.45902	70.67086	75.33328	11.46596
35	17.5	70.50631	70.51391	74.88119	13.91358
36	18	70.65466	70.45952	74.46214	16.57926
37	18.5	70.9021	70.50736	74.08275	19.44482
38	19	71.24576	70.65628	73.74891	22.48891
39	19.5	71.68195	70.90429	73.46583	25.68743
40	20	72.2061	71.24851	73.23797	29.01386
41	20.5	72.81289	71.68524	73.06906	32.43963
42	21	73.49617	72.20992	72.96208	35.93447
43	21.5	74.24911	72.8172	72.91924	39.46685
44	22	75.06419	73.50095	72.94203	43.00428
45	22.5	75.9333	74.25431	73.03114	46.51376
46	23	76.84778	75.06977	73.18652	49.96209
47	23.5	77.79854	75.9392	73.4074	53.31631
48	24	78.7761	76.85396	73.69226	56.54401
49	24.5	79.77071	77.80493	74.03889	59.61374
50	25	80.77242	78.78264	74.4444	62.4953
51	25.5	81.77121	79.77734	74.90526	65.16011
52	26	82.75704	80.77909	75.41734	67.5815
53	26.5	83.71997	81.77784	75.97595	69.73501
54	27	84.65028	82.76357	76.5759	71.59865
55	27.5	85.53852	83.72634	77.21153	73.15316
56	28	86.37565	84.65642	77.87679	74.38218
57	28.5	87.15308	85.54438	78.56528	75.27249
58	29	87.86282	86.38116	79.27034	75.81414
59	29.5	88.49748	87.15819	79.98512	76.00053
60	30	89.05044	87.86747	80.70261	75.82856
61	30.5	89.51582	88.50163	81.41573	75.29865
62	31	89.88863	89.05404	82.11744	74.41473
63	31.5	90.16477	89.51884	82.80074	73.18428
64	32	90.34107	89.89104	83.45881	71.6182
65	32.5	90.41536	90.16654	84.08503	69.73078
66	33	90.38648	90.34218	84.67306	67.53954
67	33.5	90.25426	90.4158	85.21692	65.06508
68	34	90.01961	90.38623	85.71105	62.33091
69	34.5	89.68442	90.25333	86.15034	59.36318
70	35	89.2516	90.01801	86.53021	56.19048
71	35.5	88.72506	89.68215	86.84666	52.84355

72	36	88.10963	89.2487	87.09628	49.35497
73	36.5	87.41107	88.72154	87.27634	45.75887
74	37	86.63597	88.10552	87.38475	42.09058
75	37.5	85.79173	87.40642	87.42016	38.38663
76	38	84.88644	86.63082	87.38191	34.68271
77	38.5	83.92887	85.78613	87.27009	31.01666
78	39	82.9283	84.88045	87.08551	27.42476
79	39.5	81.89451	83.92253	86.82971	23.94304
80	40	80.83765	82.92169	86.50494	20.60656
81	40.5	79.76811	81.88769	86.11415	17.44911
82	41	78.69648	80.83068	85.66096	14.50281
83	41.5	77.6334	79.76106	85.14962	11.79785
84	42	76.58949	78.68942	84.58496	9.362092
85	42.5	75.57518	77.62641	83.97239	7.220878
86	43	74.6007	76.58263	83.31781	5.396707
87	43.5	73.67588	75.56852	82.62754	3.909023
88	44	72.81013	74.5943	81.9083	2.774002
89	44.5	72.0123	73.66982	81.16713	2.004381
90	45	71.2906	72.80447	80.4113	1.609318
91	45.5	70.65251	72.00709	79.64827	1.594282
92	46	70.10473	71.28589	78.88559	1.960989
93	46.5	69.65307	70.64836	78.13086	2.707364
94	47	69.30242	70.10118	77.39159	3.827551
95	47.5	69.05668	69.65016	76.67522	5.311953
96	48	68.91872	69.30017	75.98895	7.147312
97	48.5	68.89035	69.05512	75.33973	9.316825
98	49	68.97232	68.91787	74.73417	11.8003
99	49.5	69.16426	68.89023	74.17846	14.57434
100	50	69.46471	68.97293	73.67835	17.61257
101	50.5	69.87113	69.16559	73.23902	20.88586
102	51	70.37993	69.46675	72.8651	24.36264
103	51.5	70.98646	69.87387	72.56057	28.00918
104	52	71.68509	70.38334	72.32875	31.78994
105	52.5	72.46926	70.99051	72.17224	35.66787
106	53	73.33152	71.68974	72.0929	39.60483
107	53.5	74.26363	72.47447	72.09185	43.56191
108	54	75.2566	73.33725	72.16941	47.49987
109	54.5	76.30081	74.26981	72.32513	51.37947
110	55	77.3861	75.26317	72.5578	55.1619
111	55.5	78.50184	76.30772	72.86542	58.80916
112	56	79.63706	77.39327	73.24522	62.28444
113	56.5	80.78056	78.5092	73.69375	65.55248
114	57	81.92101	79.64455	74.20681	68.57993
115	57.5	83.04705	80.7881	74.77958	71.33571
116	58	84.14744	81.92852	75.4066	73.7913
117	58.5	85.21112	83.05446	76.08187	75.92106
118	59	86.22737	84.15467	76.79887	77.70247
119	59.5	87.18589	85.2181	77.55064	79.11639
120	60	88.07692	86.23403	78.32986	80.14726

**EX Listing (10-3)**

'These functions must be modified for actual problem usage

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## Proportional and Integral Controllers

```
'prm(1) -rhoVCp -kJ / C
'prm(2) -Tis -C
'prm(3) -taud -Min
'prm(4) -Kc -b kJ/min-C
'prm(5) -WCp -kJ / Min - C
'prm(6) -Tr -C
'prm(7) -taum -Min
'prm(8) -tauI -Min
```

```
'x - time
'y1 - T
'y2 - To
'y3 - Tm
'y4 - errsum
```

Public Function fff1(x, y1, y2, y3, y4, y5, prm)

```
step = 1
If x < 10 Then
    step = 0
End If
```

```
ti = prm(2) + step * (-20)
```

```
qs = prm(5) * (prm(6) - prm(2))
q = qs + prm(4) * (prm(6) - y3) + prm(4) * y4 / prm(8)
dTdt = (prm(5) * (ti - y1) + q) / prm(1)
```

```
fff1 = dTdt
End Function
```

Public Function fff2(x, y1, y2, y3, y4, y5, prm)

```
step = 1
If x < 10 Then
    step = 0
End If
```

```
ti = prm(2) + step * (-20)
```

```
qs = prm(5) * (prm(6) - prm(2))
q = qs + prm(4) * (prm(6) - y3) + prm(4) * y4 / prm(8)
dTdt = (prm(5) * (ti - y1) + q) / prm(1)
```

```
fff2 = (y1 - y2 - (prm(3) / 2) * dTdt) * 2 / prm(3)
```

```
End Function
```

Public Function fff3(x, y1, y2, y3, y4, y5, prm)

$$\text{fff3} = (y2 - y3) / \text{prm}(7)$$

End Function

Public Function fff4(x, y1, y2, y3, y4, y5, prm)

$$\text{fff4} = \text{prm}(6) - y3$$

End Function

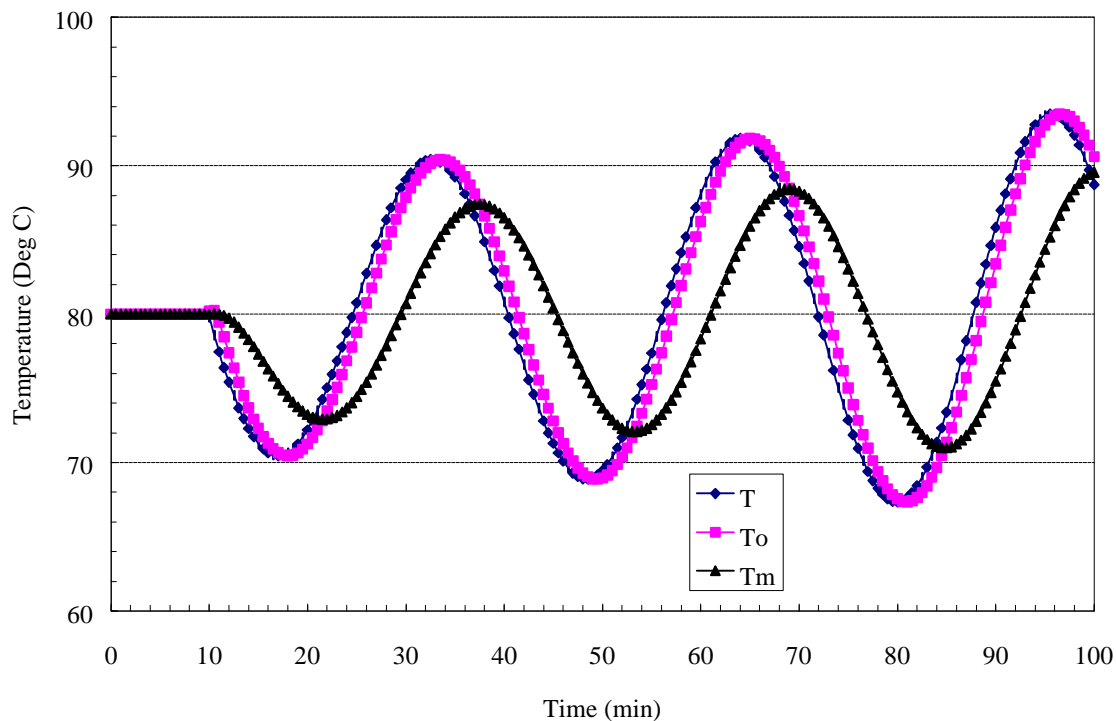
Public Function fff5(x, y1, y2, y3, y4, y5, prm)

$$\text{fff5} = 0$$

End Function

### EX Figure (10-3)

Dynamics of a Heated Tank - PI Control -  $K_c = 500$



(d)

The spreadsheet resulting from P control alone with  $K_c = 500$  is given by EX Spreadsheet (10-4). The user defined functions for fff1...fff2 are given by EX Listing (10-4). The plot is given in EX Figure (10-4).

**EX Spreadsheet (10-4)**

Problem 10(d)  
Dynamics of a Heated Tank with Proportional Temperature Control

Parameters	Value	Index	<-----Integration----->				
			Time (min)	T	To	Tm	errsum
rhoVc <sub>p</sub> - kJ/C	4000	0	0	80	80	80	0
T <sub>is</sub> - C	60	1	0.5	80	80	80	0
taud - min	1	2	1	80	80	80	0
K <sub>c</sub> - kJ/min-C	500	3	1.5	80	80	80	0
WC <sub>p</sub> - kJ/min-C	500	4	2	80	80	80	0
Tr - C	80	5	2.5	80	80	80	0
taum - min	5	6	3	80	80	80	0
taul - min	2	7	3.5	80	80	80	0
Integration Increment - h	0.5	8	4	80	80	80	0
		9	4.5	80	80	80	0
		10	5	80	80	80	0
		11	5.5	80	80	80	0
		12	6	80	80	80	0
		13	6.5	80	80	80	0
		14	7	80	80	80	0
		15	7.5	80	80	80	0
		16	8	80	80	80	0
		17	8.5	80	80	80	0
		18	9	80	80	80	0
		19	9.5	80	80	80	0
		20	10	79.79167	80.20833	80	0
		21	10.5	78.59167	80.23837	80.03409	-0.00726
		22	11	77.46327	79.48226	80.02139	-0.02376
		23	11.5	76.40651	78.47555	79.92269	-0.01339
		24	12	75.42252	77.41926	79.73402	0.068882
		25	12.5	74.51212	76.38969	79.46362	0.266329
		26	13	73.67547	75.41618	79.12352	0.61691
		27	13.5	72.91192	74.51004	78.72634	1.152325
		28	14	72.22013	73.67527	78.28415	1.898064
		29	14.5	71.59813	72.91269	77.8081	2.873804
		30	15	71.04344	72.22147	77.30824	4.093917
		31	15.5	70.55313	71.59986	76.79356	5.568018
		32	16	70.12397	71.04545	76.27197	7.301495
		33	16.5	69.75244	70.55536	75.7504	9.29603
		34	17	69.43486	70.12634	75.2348	11.55009
		35	17.5	69.16745	69.75491	74.73025	14.05938
		36	18	68.94637	69.43739	74.24102	16.81727
		37	18.5	68.76775	69.17	73.77057	19.81522
		38	19	68.62782	68.9489	73.32172	23.04309
		39	19.5	68.52284	68.77025	72.89663	26.48953
		40	20	68.4492	68.63025	72.49689	30.14223
		41	20.5	68.40344	68.52519	72.12358	33.98823
		42	21	68.38225	68.45146	71.77735	38.01413
		43	21.5	68.38248	68.40559	71.45842	42.20632
		44	22	68.40118	68.38427	71.16669	46.55117
		45	22.5	68.43561	68.38438	70.90176	51.03516
		46	23	68.48319	68.40295	70.66296	55.64505
		47	23.5	68.54159	68.43724	70.44944	60.36798
		48	24	68.60863	68.4847	70.26015	65.19157
		49	24.5	68.68237	68.54296	70.09394	70.10398
		50	25	68.76104	68.60987	69.94952	75.094
		51	25.5	68.84305	68.68348	69.82555	80.15106

52	26	68.92702	68.76203	69.72063	85.26527
53	26.5	69.01171	68.84393	69.63335	90.42748
54	27	69.09606	68.92779	69.56227	95.62922
55	27.5	69.17914	69.01238	69.50599	100.8627
56	28	69.26018	69.09663	69.46309	106.121
57	28.5	69.33854	69.17962	69.43225	111.3976
58	29	69.41368	69.26058	69.41214	116.687
59	29.5	69.48519	69.33885	69.40153	121.9839
60	30	69.55275	69.41393	69.39922	127.2841
61	30.5	69.61613	69.48537	69.40411	132.5835
62	31	69.67518	69.55288	69.41514	137.8789
63	31.5	69.72982	69.61621	69.43134	143.1675
64	32	69.78004	69.67522	69.45183	148.4469
65	32.5	69.82586	69.72982	69.47576	153.7151
66	33	69.86737	69.78	69.5024	158.9707
67	33.5	69.90468	69.8258	69.53107	164.2124
68	34	69.93795	69.86728	69.56116	169.4393
69	34.5	69.96736	69.90458	69.59212	174.6511
70	35	69.99309	69.93783	69.6235	179.8472
71	35.5	70.01536	69.96722	69.65486	185.0276
72	36	70.03439	69.99294	69.68586	190.1924
73	36.5	70.0504	70.01521	69.71619	195.3418
74	37	70.06364	70.03423	69.74559	200.4763
75	37.5	70.07433	70.05025	69.77385	205.5964
76	38	70.08269	70.06349	69.80082	210.7027
77	38.5	70.08896	70.07418	69.82635	215.7958
78	39	70.09335	70.08255	69.85036	220.8766
79	39.5	70.09607	70.08882	69.87277	225.9457
80	40	70.09731	70.09321	69.89356	231.0041
81	40.5	70.09727	70.09594	69.9127	236.0525
82	41	70.09612	70.09719	69.93021	241.0917
83	41.5	70.09403	70.09715	69.94611	246.1225
84	42	70.09115	70.09601	69.96044	251.1458
85	42.5	70.08763	70.09393	69.97324	256.1623
86	43	70.08358	70.09106	69.9846	261.1728
87	43.5	70.07914	70.08754	69.99456	266.178
88	44	70.07441	70.08351	70.00322	271.1785
89	44.5	70.06947	70.07907	70.01065	276.175
90	45	70.06442	70.07435	70.01693	281.168
91	45.5	70.05932	70.06942	70.02216	286.1582
92	46	70.05425	70.06437	70.02641	291.146
93	46.5	70.04926	70.05928	70.02978	296.1319
94	47	70.04439	70.05422	70.03234	301.1164
95	47.5	70.03968	70.04923	70.03418	306.0997
96	48	70.03516	70.04436	70.03538	311.0823
97	48.5	70.03087	70.03966	70.036	316.0644
98	49	70.02681	70.03515	70.03613	321.0464
99	49.5	70.023	70.03086	70.03583	326.0284
100	50	70.01946	70.0268	70.03516	331.0106
101	50.5	70.01618	70.023	70.03418	335.9933
102	51	70.01316	70.01946	70.03294	340.9765
103	51.5	70.01041	70.01618	70.03149	345.9604
104	52	70.00792	70.01317	70.02989	350.945
105	52.5	70.00568	70.01042	70.02816	355.9305
106	53	70.00369	70.00793	70.02635	360.9169
107	53.5	70.00193	70.00569	70.02449	365.9042
108	54	70.00038	70.0037	70.0226	370.8924
109	54.5	69.99905	70.00193	70.02072	375.8816
110	55	69.99791	70.00039	70.01885	380.8717
111	55.5	69.99695	69.99906	70.01703	385.8627
112	56	69.99616	69.99792	70.01526	390.8546

113	56.5	69.99552	69.99696	70.01356	395.8474
114	57	69.99502	69.99617	70.01194	400.8411
115	57.5	69.99464	69.99553	70.01041	405.8355
116	58	69.99438	69.99503	70.00897	410.8306
117	58.5	69.99422	69.99465	70.00762	415.8265
118	59	69.99415	69.99439	70.00637	420.823
119	59.5	69.99415	69.99423	70.00523	425.8201
120	60	69.99422	69.99415	70.00417	430.8177

**EX Listing (10-4)**

'These functions must be modified for actual problem usage

'Written by EMRosen 6/19/97

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'Proportional Control

'prm(1) -rhoVCp -kJ / C

'prm(2) -Tis -C

'prm(3) -taud -Min

'prm(4) -Kc -b kJ/min-C

'prm(5) -WCp -kJ / Min - C

'prm(6) -Tr -C

'prm(7) -taum -Min

'prm(8) -tauI -Min

'x - time

'y1 -T

'y2 - To

'y3 - Tm

'y4 - errsum

Public Function fff1(x, y1, y2, y3, y4, y5, prm)

step = 1

If x < 10 Then

    step = 0

End If

ti = prm(2) + step \* (-20)

qs = prm(5) \* (prm(6) - prm(2))

```

q = qs + prm(4) * (prm(6) - y3)
dTdt = (prm(5) * (ti - y1) + q) / prm(1)

```

```

fff1 = dTdt
End Function

```

```

Public Function fff2(x, y1, y2, y3, y4, y5, prm)

```

```

step = 1
If x < 10 Then
    step = 0
End If

```

```

ti = prm(2) + step * (-20)

```

```

qs = prm(5) * (prm(6) - prm(2))
q = qs + prm(4) * (prm(6) - y3)
dTdt = (prm(5) * (ti - y1) + q) / prm(1)

```

```

fff2 = (y1 - y2 - (prm(3) / 2) * dTdt) * 2 / prm(3)

```

```

End Function

```

```

Public Function fff3(x, y1, y2, y3, y4, y5, prm)

```

```

fff3 = (y2 - y3) / prm(7)

```

```

End Function

```

```

Public Function fff4(x, y1, y2, y3, y4, y5, prm)

```

```

fff4 = prm(6) - y3

```

```

End Function

```

```

Public Function fff5(x, y1, y2, y3, y4, y5, prm)

```

```

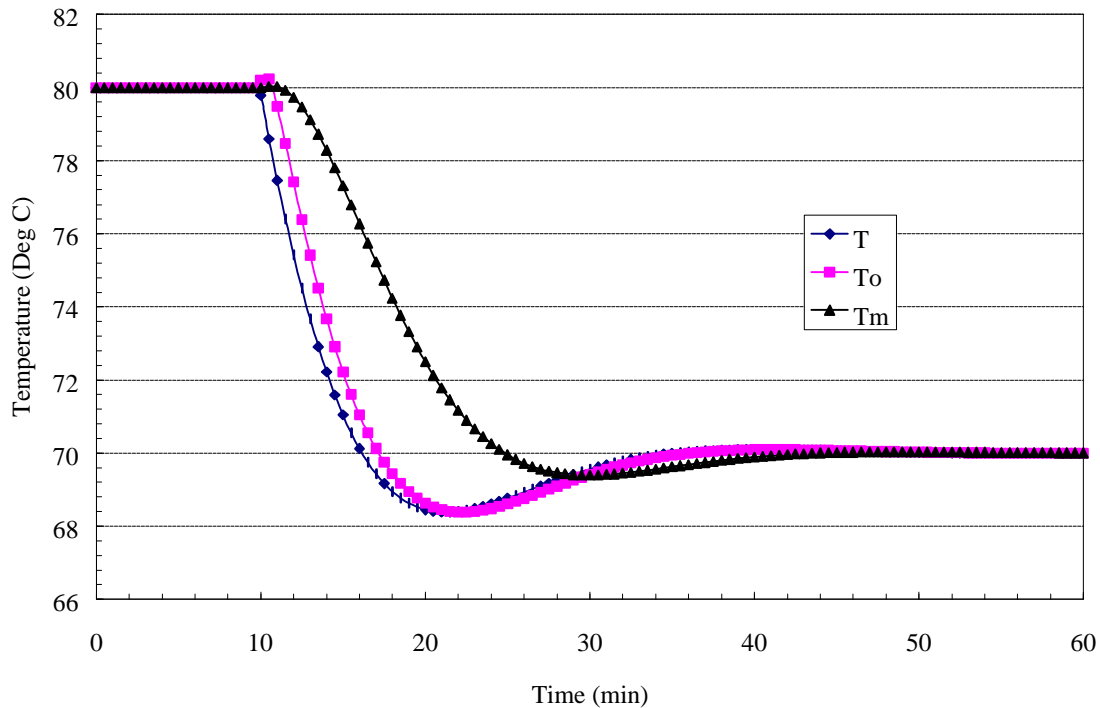
fff5 = 0

```

```

End Function

```

**EX Figure (10-4)**Dynamics of a Heated Tank - Proportional Control Only with  $K_c = 500$ **(e)**

The spreadsheet resulting from P control alone for  $K_c = 5000$  with a maximum limit on  $q$  (equal to  $2.6 \times 10000$ ) and a minimum on  $q$  (equal to 0) is given by EX Spreadsheet (10-5). The user defined functions for  $fff1 \dots fff2$  are given by EX Listing (10-5). The plots are given in EX Figure (10-5) and EX Figure (10-6).

**EX Spreadsheet (10-5)**

Problem 10(e)

Dynamics of a Heated Tank with P Temperature Control and Limits on  $q$ 

Parameters	Value	Index	Time(min)	T	To	Tm	errsum	q	qlim
rhoVCp - kJ/C	4000	0	0	80	80	80	0	10000	10000
Tis - C	60	1	0.5	80	80	80	0	10000	10000
taud - min	1	2	1	80	80	80	0	10000	10000
Kc - kJ/min-C	5000	3	1.5	80	80	80	0	10000	10000
WCp - kJ/min-C	500	4	2	80	80	80	0	10000	10000
Tr - C	80	5	2.5	80	80	80	0	10000	10000
taum - min	5	6	3	80	80	80	0	10000	10000
taul - min	2	7	3.5	80	80	80	0	10000	10000
		8	4	80	80	80	0	10000	10000
Integration Increment - h	0.5	9	4.5	80	80	80	0	10000	10000
		10	5	80	80	80	0	10000	10000
		11	5.5	80	80	80	0	10000	10000

12	6	80	80	80	0	10000	10000
13	6.5	80	80	80	0	10000	10000
14	7	80	80	80	0	10000	10000
15	7.5	80	80	80	0	10000	10000
16	8	80	80	80	0	10000	10000
17	8.5	80	80	80	0	10000	10000
18	9	80	80	80	0	10000	10000
19	9.5	80	80	80	0	10000	10000
20	10	80.33333	79.66667	80	0.833333	60000	26000
21	10.5	82.25192	79.61884	79.94551	5.844945	60272.458	26000
22	11	84.05426	80.82817	79.96586	10.8713	60170.695	26000
23	11.5	85.74741	82.43457	80.12355	15.85471	59382.231	26000
24	12	87.33797	84.12002	80.42477	20.7233	57876.157	26000
25	12.5	88.83217	85.76949	80.85663	25.408	55716.87	26000
26	13	90.23584	87.34384	81.40094	29.84789	52995.32	26000
27	13.5	91.55446	88.8321	82.03915	33.99141	49804.229	26000
28	14	92.79319	90.23368	82.75406	37.79598	46229.681	26000
29	14.5	93.95688	91.55165	83.53025	41.22716	42348.771	26000
30	15	95.05005	92.79026	84.35411	44.25781	38229.439	26000
31	15.5	96.077	93.95401	85.21379	46.86711	33931.051	26000
32	16	97.04172	95.04732	86.09896	49.03979	29505.19	26000
33	16.5	97.92717	96.09524	87.00071	50.7654	24996.474	24996.47
34	17	98.37738	97.20401	87.91915	52.03571	20404.257	20404.26
35	17.5	98.24027	98.04357	88.84616	52.84384	15769.178	15769.18
36	18	97.55701	98.4029	89.74281	53.19431	11285.935	11285.93
37	18.5	96.39319	98.22148	90.5624	53.11383	7188.0172	7188.017
38	19	94.83836	97.51179	91.2609	52.65216	3695.4806	3695.481
39	19.5	93.00112	96.33039	91.80202	51.87929	989.91381	989.9138
40	20	91.02462	94.74474	92.15984	50.88092	-799.2141	0
41	20.5	89.14494	92.87474	92.31683	49.75353	-1584.158	0
42	21	87.37913	90.97111	92.27779	48.59718	-1388.939	0
43	21.5	85.72032	89.12773	92.06369	47.50505	-318.4445	0
44	22	84.2346	87.33604	91.69759	46.55886	1512.0335	1512.033
45	22.5	83.09616	85.63582	91.199	45.82979	4004.9882	4004.988
46	23	82.36217	84.18031	90.59687	45.37731	7015.6543	7015.654
47	23.5	82.0595	83.06569	89.92938	45.2439	10353.076	10353.08
48	24	82.1889	82.35151	89.23836	45.45188	13808.181	13808.18
49	24.5	82.72554	82.06685	88.5658	46.00245	17171.02	17171.02
50	25	83.62132	82.21255	87.95106	46.87639	20244.709	20244.71
51	25.5	84.80862	82.76326	87.42857	48.03593	22857.135	22857.14
52	26	86.20506	83.67024	87.02592	49.42774	24870.421	24870.42
53	26.5	87.71525	84.86909	86.76243	50.9867	26187.868	26000
54	27	89.18659	86.29173	86.64953	52.6399	26752.365	26000
55	27.5	90.56878	87.76639	86.6867	54.31176	26566.488	26000
56	28	91.86119	89.20957	86.85937	55.93049	25703.168	25703.17
57	28.5	92.96145	90.62188	87.1517	57.43231	24241.512	24241.51
58	29	93.78573	91.91414	87.54574	58.76165	22271.289	22271.29
59	29.5	94.29668	92.99608	88.01571	59.87382	19921.457	19921.46
60	30	94.47642	93.80532	88.53113	60.73832	17344.349	17344.35
61	30.5	94.32736	94.3022	89.05963	61.34049	14701.833	14701.83
62	31	93.87135	94.46862	89.5692	61.68186	12153.985	12153.98
63	31.5	93.14772	94.30742	90.03017	61.77944	9849.1553	9849.155
64	32	92.2102	93.841	90.41691	61.66413	7915.4276	7915.428
65	32.5	91.1232	93.10919	90.70926	61.37834	6453.7099	6453.71
66	33	89.95752	92.16612	90.89345	60.97299	5532.7605	5532.761
67	33.5	88.78589	91.07643	90.96273	60.50414	5186.3381	5186.338
68	34	87.67851	89.91096	90.91749	60.02944	5412.5587	5412.559
69	34.5	86.69901	88.74231	90.76492	59.60463	6175.4079	6175.408
70	35	85.90087	87.6404	90.51835	59.28027	7408.2335	7408.233
71	35.5	85.32463	86.66843	90.19621	59.09893	9018.9299	9018.93
72	36	84.996	85.87935	89.82071	59.09299	10896.44	10896.44

73	36.5	84.92488	85.3131	89.41637	59.28305	12918.135	12918.13
74	37	85.1054	84.99476	89.00848	59.67721	14957.589	14957.59
75	37.5	85.51688	84.93359	88.62154	60.27106	16892.287	16892.29
76	38	86.12567	85.12315	88.27784	61.04842	18610.784	18610.78
77	38.5	86.88766	85.54226	87.99621	61.98281	20018.926	20018.93
78	39	87.75131	86.15687	87.79104	63.0394	21044.8	21044.8
79	39.5	88.66103	86.92258	87.67157	64.17741	21642.15	21642.15
80	40	89.56069	87.78771	87.64157	65.35284	21792.134	21792.13
81	40.5	90.39694	88.6967	87.69933	66.52114	21503.373	21503.37
82	41	91.12231	89.59351	87.83793	67.63997	20810.354	20810.35
83	41.5	91.69784	90.42506	88.04593	68.67159	19770.357	19770.36
84	42	92.09505	91.14424	88.30817	69.58496	18459.138	18459.14
85	42.5	92.2972	91.71252	88.60687	70.35731	16965.673	16965.67
86	43	92.2999	92.10188	88.92273	70.97522	15386.331	15386.33
87	43.5	92.11081	92.29609	89.23624	71.43498	13818.822	13818.82
88	44	91.74877	92.29121	89.52874	71.7425	12356.31	12356.31
89	44.5	91.2422	92.09538	89.78359	71.91255	11082.026	11082.03
90	45	90.62704	91.72777	89.98706	71.96752	10064.689	10064.69
91	45.5	89.9443	91.21711	90.12901	71.93579	9354.9687	9354.969
92	46	89.2374	90.59952	90.20337	71.84983	8983.1697	8983.17
93	46.5	88.54954	89.91608	90.20836	71.74405	8958.2217	8958.222
94	47	87.92104	89.21021	90.1464	71.65269	9267.9854	9267.985
95	47.5	87.38712	88.52495	90.02384	71.6078	9880.8062	9880.806
96	48	86.97594	87.90042	89.85037	71.63738	10748.171	10748.17
97	48.5	86.70725	87.37154	89.63835	71.76389	11808.269	11808.27
98	49	86.5915	86.96615	89.40196	72.00311	12990.207	12990.21
99	49.5	86.62959	86.70359	89.15628	72.36348	14218.597	14218.6
100	50	86.8132	86.59395	88.91635	72.84587	15418.242	15418.24
101	50.5	87.12564	86.63777	88.69628	73.4438	16518.615	16518.62
102	51	87.5432	86.82639	88.50842	74.14419	17457.891	17457.89
103	51.5	88.03679	87.14287	88.36274	74.92832	18186.295	18186.3
104	52	88.57395	87.56326	88.26628	75.7732	18668.606	18668.61
105	52.5	89.12088	88.05839	88.22286	76.65315	18885.684	18885.68
106	53	89.64454	88.59573	88.233	77.54137	18834.986	18834.99
107	53.5	90.11454	89.14154	88.29399	78.41163	18530.061	18530.06
108	54	90.50494	89.66287	88.40018	79.23981	17999.114	17999.11
109	54.5	90.79553	90.12956	88.54345	80.00524	17282.739	17282.74
110	55	90.97288	90.51586	88.7138	80.69182	16431.012	16431.01
111	55.5	91.03086	90.80188	88.89997	81.28879	15500.127	15500.13
112	56	90.97068	90.97445	89.09024	81.79117	14548.797	14548.8
113	56.5	90.80058	91.02774	89.27307	82.1998	13634.65	13634.65
114	57	90.53505	90.96325	89.43783	82.52112	12810.826	12810.83
115	57.5	90.19369	90.78945	89.5754	82.76652	12122.986	12122.99
116	58	89.79988	90.52103	89.67863	82.95147	11606.87	11606.87
117	58.5	89.37919	90.17773	89.74269	83.09446	11286.56	11286.56
118	59	88.95781	89.78301	89.7653	83.21573	11173.494	11173.49
119	59.5	88.56094	89.36245	89.74674	83.33605	11266.281	11266.28
120	60	88.21132	88.9422	89.68974	83.47543	11551.291	11551.29

## EX Listing (10-5)

These functions must be modified for actual problem usage

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## Proportional Control and Limits on q

```
'prm(1) -rhoVCp -kJ / C
'prm(2) -Tis -C
'prm(3) -taud -Min
'prm(4) -Kc -b kJ/min-C
'prm(5) -WCp -kJ / Min - C
'prm(6) -Tr -C
'prm(7) -taum -Min
'prm(8) -tauI -Min
```

```
'x - time
'y1 -T
'y2 - To
'y3 - Tm
'y4 - errsum
```

Public Function fff1(x, y1, y2, y3, y4, y5, prm)

```
ti = prm(2)
```

```
step = 1
If x < 10 Then
    step = 0
End If
```

```
tr = prm(6) + step * 10
```

```
qs = prm(5) * (prm(6) - prm(2))
q = qs + prm(4) * (tr - y3)
```

```
qlim = q
```

```
If q < 0 Then
    qlim = 0
End If
```

```
If q > 26000 Then
    qlim = 26000
End If
```

```
dTdt = (prm(5) * (ti - y1) + qlim) / prm(1)
```

```
fff1 = dTdt
End Function
```

Public Function fff2(x, y1, y2, y3, y4, y5, prm)

```
ti = prm(2)
```

```
step = 1
```

```
If x < 10 Then
    step = 0
End If
```

```
tr = prm(6) + step * 10
```

```
qs = prm(5) * (prm(6) - prm(2))
q = qs + prm(4) * (tr - y3)
```

```
qlim = q
```

```
If q < 0 Then
    qlim = 0
End If
```

```
If q > 26000 Then
    qlim = 26000
End If
```

```
dTdt = (prm(5) * (ti - y1) + qlim) / prm(1)
```

```
fff2 = (y1 - y2 - (prm(3) / 2) * dTdt) * 2 / prm(3)
```

```
End Function
```

```
Public Function fff3(x, y1, y2, y3, y4, y5, prm)
```

```
fff3 = (y2 - y3) / prm(7)
```

```
End Function
```

```
Public Function fff4(x, y1, y2, y3, y4, y5, prm)
```

```
step = 1
If x < 10 Then
    step = 0
End If
```

```
tr = prm(6) + step * 10
fff4 = tr - y3
```

```
End Function
```

```
Public Function fff5(x, y1, y2, y3, y4, y5, prm)
```

```
fff5 = 0
```

```
End Function
```

```
Public Function qqlim(x, y1, y2, y3, y4, y5, prm, nr)
```

```
'nr = 1 gives q
'nr = 2 gives qlim
```

```
ti = prm(2)
step = 1
If x < 10 Then
    step = 0
End If

tr = prm(6) + step * 10

qs = prm(5) * (prm(6) - prm(2))
q = qs + prm(4) * (tr - y3)

qlim = q

If q < 0 Then
    qlim = 0
End If

If q > 26000 Then
    qlim = 26000
End If

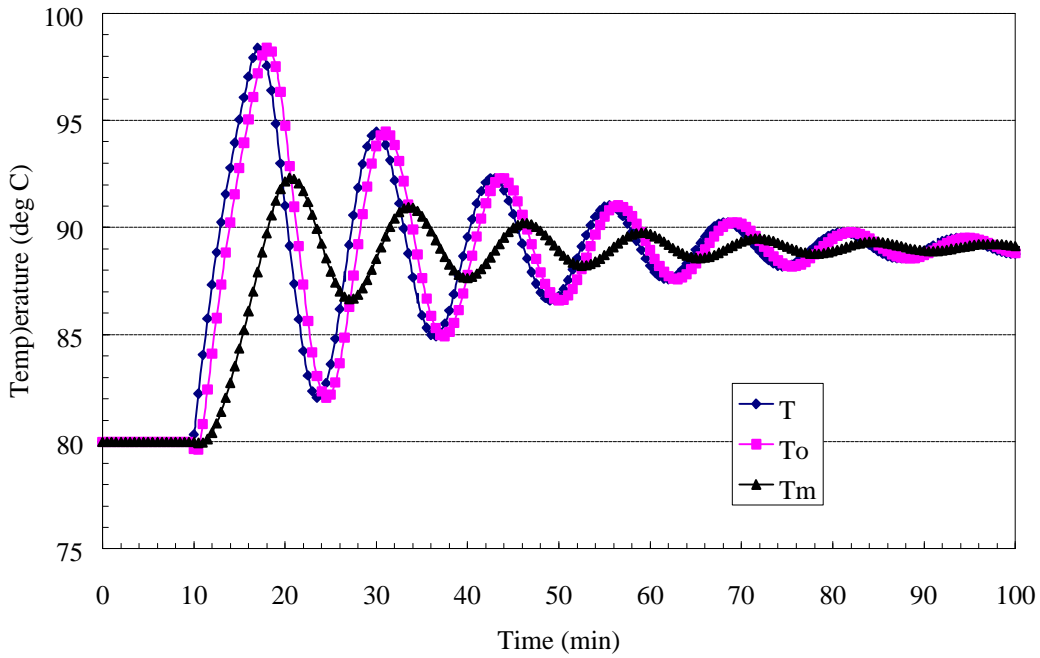
If nr = 1 Then
    qqlim = q
    Exit Function
End If

If nr = 2 Then
    qqlim = qlim
    Exit Function
End If

End Function
```

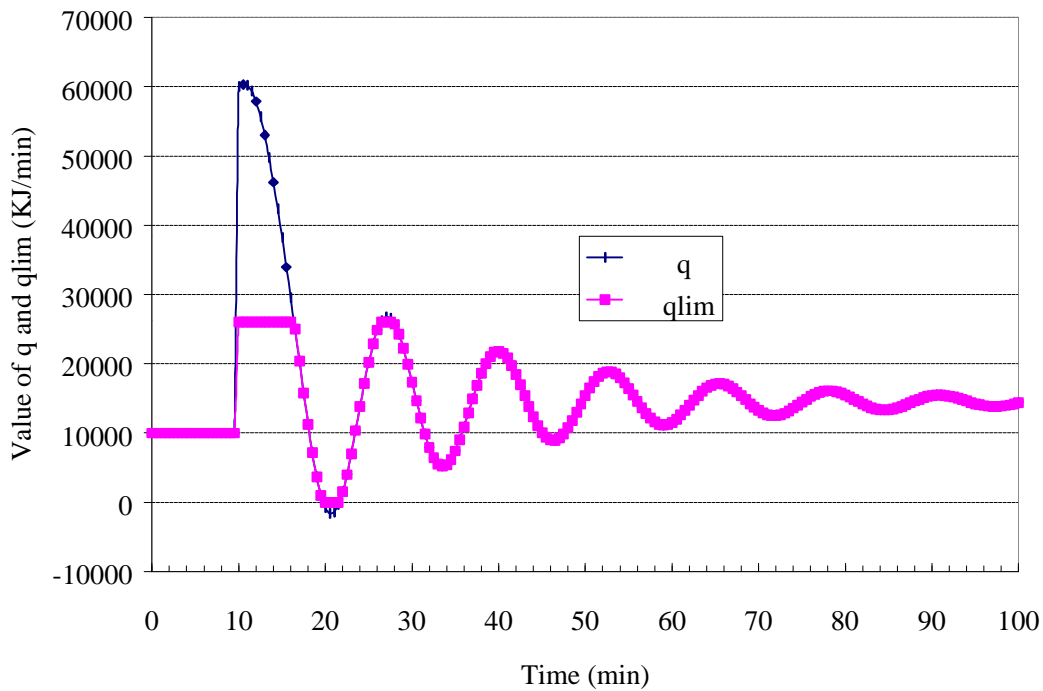
**EX Figure (10-5)**

Dynamics of a Heat Tank for P Control with Limits on  $q$



**EX Figure (10-6)**

Dynamics of a Heated Tank for P Control with Limits on  $q$



## References

1. Cutlip, M. B., J. J. Hwalek, H. E. Nuttall and M. Shacham, "The Use of Mathematical Software Packages in Chemical Engineering" Presented at the Chemical Engineering Summer School at Snowbird Utah, August 13, 1997.
2. Cutlip, M. B. and M. Shacham, "Polymath Solutions to the Chemical Engineering Problem Set" Presented at the Chemical Engineering Summer School at Snowbird, Utah, August 13, 1997.